Conjunctive strengthening more broadly

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Disjunctive sentences can, in certain cases, convey strengthened conjunctive inferences, a phenomenon that also arises with indefinites. This behavior has been captured on the grammatical theory of exhaustification and alternatives, on which strengthening is modulated by what alternatives are generated by grammar. We examine a series of predictions of the theory stemming from one grammatical property shared by disjunction and indefinites: their ability to undergo scope shift in certain configurations in which their alternatives – conjunction and universal quantifiers – cannot. We show that, in such cases, the absence of corresponding conjunctive and universal alternatives may license exhaustification to generate conjunctive inferences. We present evidence that this indeed occurs, building on and extending earlier observations of Santorio 2018, 2020 and Bar-Lev and Fox 2020.

1 Conjunctive strengthening

Disjunctive sentences can sometimes give rise to conjunctive inferences. This is exemplified in (1): the sentence in which disjunction occurs in the scope of an existential modal conveys the conjunctive inference that Jo may talk to Gal and that she may talk to Tal – an inference commonly referred to also as the 'free choice inference' (e.g., Kamp 1973, Zimmermann 2000, Geurts 2005, Fox 2007).

(1) Jo may talk to Gal or Tal.

Conjunctive inference: $\Diamond$(Jo talks to Gal) $\wedge$ $\Diamond$(Jo talks to Tal)

Sentences with indefinites can likewise give rise to conjunctive inferences. This is exemplified in (2): the sentence conveys the conjunctive inference corresponding to the conjunction of sentences obtained by substituting the indefinite with specific individual names (e.g., Horn 1972, Ladusaw 1979, Kadmon and Landman 1993, Dayal 1998). (While this interpretation is often formalized as universal quantification over the relevant domain – in this case, students – and referred to as a 'free choice' or 'universal inference', we retain the term 'conjunctive inference' here for uniformity.)

(2) Jo may talk to a(ny) student.

Conjunctive inference: $\bigwedge_{x \in [\![\text{student}]\!]} \Diamond(\text{Jo talks to } x)$

$\big($*a.k.a. Universal inference:* $\forall$x: student x $\rightarrow \Diamond$(Jo talks to x)$\big)$

In contrast, simple disjunctive and indefinite sentences, such as those in (3) and (4), not only fail to give rise to such inferences, but instead tend to trigger their negation. These inferences are commonly referred to as 'exclusive inference' and '*not all* inference', respectively.

(3) Jo talked to Gal or Tal.

Negation of the conjunctive inference: $\neg$(Jo talked to Gal $\wedge$ Jo talked to Tal)

(4) Jo talked to a student.

Negation of the conjunctive inference: $\neg \bigwedge_{x \in [\![\text{student}]\!]}(\text{Jo talked to } x)$

This complex behavior of disjunction and indefinites seems *prima facie* paradoxical.

Formal alternatives. Fox 2007 shows that this behavior can be explained by recourse to exhaustification in grammar, an operation that quantifies over formal alternatives to sentences and strengthens their meaning. In the case of (1) and (2), this strengthening yields the conjunctive inferences – a process we, accordingly, refer to as 'conjunctive strengthening'. Furthermore, Fox shows that recourse to formal alternatives alone already suffices for describing the complex behavior of disjunction and indefinites. In order to state this description, however, we must first define the notion of a formal alternative. Formal alternatives to a sentence are syntactic objects that are at most as complex as the sentence itself; they can be derived from the original sentence through a series of substitutions or deletions (see Katzir 2007, Fox and Katzir 2011 for details, qualifications, and motivation). It is natural to assume that such alternatives are generated in grammar: if an object is related to a sentence by means of appropriate deletions and substitutions, but cannot be generated in grammar, it is not a valid alternative. In this paper, we hone-in on some implications of this grammatical constraint on alternative generation, which we underline in the characterization in (5).

(5) **Formal alternatives:**

ALT(S) = {S′ | S′ is an object generated in grammar that can be derived from S by deletions and by substitutions of constituents with elements from the lexicon of the same category}

(Non-)closure under conjunction. A key diagnostic for distinguishing between disjunctive and indefinite sentences that give rise to conjunctive inferences vs. those that do not involves the structure of their sets of formal alternatives. Specifically, conjunctive strengthening is blocked when the set of alternatives to the sentences is closed under conjunction, but may be possible when it is not, as stated in (C) below (Fox 2007; see also Chemla 2009, Franke 2011, Singh et al. 2016, among others). (While this condition provides a good approximation of the necessary condition on the distribution of conjunctive inferences for our purposes, it can be further sharpened, as we discuss in Sect. 5.1.)

(C) Condition of non-closure under conjunction (cf. Fox 2007):

Conjunctive strengthening of a disjunctive sentence or a sentence with an indefinite, S, may obtain only if the set of formal alternatives to it, ALT(S), is <u>not closed</u> under conjunction.

Let's see how the condition applies to the sentences above. The existential modal sentence in (1) induces the set of alternatives in (6), which consists of the initial sentence (alternative 1), of two disjunct alternatives (alternatives 2 and 3), and of the conjunctive alternative (alternative 4). (We follow the convention of leaving out alternatives that are equivalent to those provided as well as those that are not relevant for the discussion at hand, say, *Bo may talk to Jo*.)

(6) ALT(Jo may talk to Gal or Tal) =

{Jo may talk to Gal or Tal, Jo may talk to Gal,
Jo may talk to Tal, Jo may talk to Gal and Tal}

The set in (6) is <u>not closed</u> under conjunction since the conjunction of the disjunct alternatives is not equivalent to any other alternative in the set. For example, it is not equivalent to the conjunctive alternative: the conjunction of the disjunct alternatives may be true (there is an accessible world in which Jo talks to Gal and another one in which Jo talks to Tal), while the conjunctive alternative is false (there is no accessible world in which Jo talks to both Gal and Tal). Accordingly, condition (C) correctly allows for the sentence to give rise to a conjunctive inference.

(7) **Condition (C) admits potential conjunctive inference for (1):**

ALT(Jo may talk to Gal or Tal) is <u>not closed</u> under conjunction.

$\big($esp., $[\![$Jo may talk to Gal$]\!] \wedge [\![$Jo may talk to Tal$]\!] \not\Leftrightarrow [\![$Jo may talk to Gal and Tal$]\!]\big)$

The simple indefinite sentence like (4) induces alternatives analogous to (6), under the simplifying assumption that Gal and Tal are the only relevant students. In addition to the sentence itself and its universal quantifier alternative (that is, alternatives 1 and 4), the sentence induces alternatives in which the domain of the indefinite or the universal quantifier in these two alternatives is replaced with its subdomains – these are the subdomain alternatives. Such alternatives are admitted by the characterization of alternatives in (5), assuming that quantifiers' domain variables are syntactically represented (see, e.g., von Fintel 1994, Stanley and Szabó 2000 on syntactically represented domain variables, and Chierchia 2013 for further details on the derivation of subdomain alternatives). Given our assumption that the initial domain consists only of Gal and Tal, the subdomain alternatives effectively reduce to the two disjunct alternatives discussed earlier.

(8) ALT(Jo talked to a$_{\{g,t\}}$ student) =

{Jo talked to a$_{\{g,t\}}$ student, Jo talked to a$_{\{g\}}$ student,

Jo talked to a$_{\{t\}}$ student, Jo talked to every$_{\{g,t\}}$ student}

In contrast to above, this set is <u>closed</u> under conjunction, as stated in (9): the conjunction of the two subdomain alternatives corresponds to the universal quantifier alternative, the conjunction of the initial sentence and a subdomain alternative corresponds to the subdomain alternative, etc. According to condition (C), then, sentence *Jo talked to a student* cannot generate a conjunctive inference. (See, again, Sect. 5.1 on a more sophisticated approach to condition (C).)

(9) **Condition (C) precludes potential conjunctive inference for (4):**

ALT(Jo talked to a$_{\{g,t\}}$ student) is <u>closed</u> under conjunction.

(esp., $[\![$Jo talked to a$_{\{g\}}$ student$]\!] \wedge [\![$Jo talked to a$_{\{t\}}$ student$]\!]$

$\Leftrightarrow [\![$Jo talked to every$_{\{g,t\}}$ student$]\!]$)

Corroborated prediction. Strikingly, condition (C) <u>can</u> be satisfied even in simple disjunctive and indefinite sentences: namely, in languages in which disjunctive markers lack conjunctive marker alternatives, or in which indefinites lack universal quantifier alternatives, the sets of alternatives generated by disjunctive and indefinite sentences are not closed under conjunction. This is becasue, in these cases, the conjunctions of the disjunct and subdomain alternatives are not equivalent to any alternatives generated by grammar.

(10) **Missing lexical alternatives:**

a. In a language that lacks a conjunctive marker, a simple disjunctive sentence satisfies condition (C), potentially enabling the derivation of a conjunctive inference.

b. In a language that lacks a universal quantifier, a simple indefinite sentence satisfies condition (C), potentially enabling the derivation of a conjunctive inference.

This prediction has sparked a lively research program in which numerous arguments have been brought forward in support of it across a wide range of constructions and languages – specifically, languages in which disjunctive markers lack conjunctive marker alternatives, and in which indefinites or other existential quantifiers lack universal quantifier alternatives (see, e.g., Davidson 2013, Bowler 2014, Bar-Lev and Margulis 2014, Singh et al. 2016, Bassi and Bar-Lev 2016, Tieu et al. 2016, 2017, Bar-Lev 2018, Staniszewski 2021, Jeretič 2021; but see Haslinger and Schmitt 2019 for a critical discussion of some of these proposals).

More general prediction. The prediction in (10) can be straightforwardly generalized beyond simple sentences and cases involving missing lexical alternatives: if a sentence contains disjunction or an indefinite, and lacks a parallel conjunctive or universal quantifier alternative – regardless of the underlying reason – it may give rise to a conjunctive inference, insofar as the set of alternatives to the sentence is not closed under conjunction. This generalization, summarized in (11), broadens the scope of (10) to the extent there exist sentences with disjunction or indefinites that lack parallel conjunctive or universal quantifier alternatives for reasons other than lexical gaps in the language.

(11) **Missing alternatives *simpliciter*:**

a. A sentence that dominates disjunction, ϕ[S or S'], and lacks a parallel conjunctive alternative, *ϕ[S or S'/S and S'], may satisfy condition (C), and thus may enable the derivation of a conjunctive inference of the form $[\![\phi[\text{S or S'/S}]]\!] \wedge [\![\phi[\text{S or S'/S'}]]\!]$.

b. A sentence that dominates an indefinite, ϕ[an NP], and lacks a parallel universal quantifier alternative, *ϕ[an NP/every NP], may satisfy condition (C), and thus may enable the derivation of a conjunctive inference of the form $\bigwedge_{x\in[\![\text{NP}]\!]} [\![\phi[\text{an NP}]]\!]^{[an\ NP \rightarrow x]}$.

One schema for constructing such sentences that does not hinge on lexical gaps relies on the selective applicability of certain grammatical operations: if a grammatical operation can target disjunction or an indefinite, but cannot target conjunction or a universal quantifier, then applying this operation yields sentences containing disjunction or indefinites that lack parallel conjunctive or universal quantifier alternatives. This is stated in a slightly more general form in (12):

(12) **A schematic consequence of the constraint in (5):**

If expresssion α has an alternative β, $\beta \in$ALT(α), and operation OP can apply to α in sentence S to yield a well-formed structure, OP(α, S[α]), but it cannot do so to β in corresponding sentence S in which β replaces α, *OP(β, S[α/β]), then the former, well-formed structure lacks the latter structure as an alternative, *OP(β, S[α/β]) $\notin$ ALT(OP(α, S[α])).

Preview. The proposal that alternatives are constrained as characterized in (5) can be evaluated by examining the inferences that relevant sentences give rise to.[1,2] In this paper, we explore several instances of the schema in (12), all of which exploit asymmetries in the scope-shifting abilities of

[1]Figuring out what alternatives a sentence may have, and what inferences it can accordingly give rise to, is a complex task. For example, one structure in which two scalemate items exhibit a famously complementary distribution is the existential construction, as illustrated in (i). This should in principle make existential constructions a promising empirical domain for testing the generalization in (12).

(i) a. There are some students from my class in the garden.
  b. *There are all students from my class in the garden.

Sentence (ia) can give rise to the scalar implicature that not all students from my class are in the garden. At first glance, this implicature appears to be derived on the basis of the ungrammatical (surface) alternative in (ib), potentially challenging the formulation in (5) and its consequence in (12). However, the challenge is only apparent. For example, the existential construction in (ia) plausibly has the complex structure in (iia), as argued by, e.g., Chomsky 1995, Lasnik 1995. If so, it has the well-formed alternative in (iib), whose negation yields the observed scalar implicature.

(ii) a. [some students from my class [there are ~~[some students from my class~~ in the garden]]]
  b. [all students from my class are in the garden]

[2]A reviewer draws attention to some underappreciated observations by Schmitt 2013, which may serve as potential cases in which disjunction gives rise to conjunctive inferences as a result of missing conjunctive alternatives. Two such examples are provided in (i), both of which yield a conjunctive inference (Schmitt 2013, p. 226):

(i) a. Ich esse (gern) Karotten, Bohnen oder (auch) Gurken. [German]
  'I (like to) eat carrots, beans or (also) cucumbers.'
  b. Geboten werden lange Artikel, Analysen und Abhandlungen über Geschichte, Wissenschaft und Politik. Sigmund Freud, Albert Einstein oder Bill Clinton haben für die Britannica geschrieben. [German]
  'The Encyclopedia Britannica offers in-depth articles, analysis and treatments on history, science and politics. Sigmund Freud, Albert Einstein or Bill Clinton contributed to the Britannica.'

disjunction and indefinites vs. conjunction and universal quantifiers. In Sect. 2, we examine cases of scope-taking out of islands, which is possible for disjunction and indefinites – this is their 'exceptional scope taking' ability – but not for conjunction and universal quantifiers (e.g., Fodor and Sag 1982, Rooth and Partee 1982). We focus on one class of such examples involving non-monotonic environments, for which conjunctive inferences are correctly predicted. Sect. 3 turns to examples in which disjunction and indefinites take scope out of downward-monotone environments – again in contrast to conjunction and universal quantifiers (e.g., Mayr and Spector 2012, Fleisher 2015). We highlight two classes of such examples of scope shift out of downward-monotone environments, and show that conjunctive inferences are once more correctly predicted. In Sect. 4, we turn to some of the dauntingly many predictions of the proposal. We close the paper in Sect. 5 with the discussion of conjunctive inferences in conditionals, drawing on observations of Santorio 2018, 2020, and offer some remarks on the variation in the robustness and cancellability of conjunctive inferences.

2 An asymmetry in scope taking

Disjunction and indefinites can take scope out of islands, in contrast to conjunction and universal quantifiers. Consequently, sentences in which disjunction or an indefinite undergo exceptional scope shift lack parallel conjunctive and universal quantifier alternatives, as these are not generated by grammar. This may result in their sets of alternatives not being closed under conjunction, thereby enabling the derivation of conjunctive inferences. We argue that this possibility is realized.

2.1 Exceptional scope taking and alternatives

Disjunction and indefinites allow for exceptional scope, that is, they can take scope out of islands (e.g., Fodor and Sag 1982, Rooth and Partee 1982). For illustration, the sentence in (13), in which

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An analysis of the sentences in (i) that assigns them parses in which a (covert) existential operator c-commands disjunction – thus assimilating them to sentence (1) in the main text – seems poorly motivated. Hence, an explanation that locates the source of the conjunctive inferences in the absence of conjunctive alternatives seems more promising. Such an account may also be extendable to their indefinite counterparts – specifically, the infamous examples of 'subtrigged' *any* NPs, which give rise to conjunctive inferences without, apparently, being c-commanded by appropriate operators, (ii).

(ii) Anyone that had time and patience contributed to the Britannica.

disjunction occurs in the restrictor of a universal quantifier at surface form, can convey merely that one of Gal and Tal is such that everyone who knows them is lucky. The reading can be brought out by the parenthetical “I don’t remember who.” (Exceptional scope readings may require focal stress on the disjunctive marker or the indefinite determiner, see, e.g., Schlenker 2006, Endriss 2009.)

(13) Everyone who knows Gal or Tal is lucky.
Can convey: (everyone who knows Gal is lucky) $\vee$ (everyone who knows Tal is lucky)

Similarly, the sentence in (14), in which an indefinite replaces the above disjunction, can convey merely that there is a student such that everyone who knows them is lucky. In addition to the above parenthetical, this reading can be brought out by a continuation like “namely, Gal.”

(14) Everyone who knows a student is lucky.
Can convey: $\exists$x: student x $\wedge$ (everyone who knows student x is lucky)

In contrast, conjunction and universal quantifiers do not allow for such exceptional scope (cf., e.g., Fodor and Sag 1982, Rooth and Partee 1982, May 1985). For illustration, a wide-scope construal of conjunction and a universal quantifier in (15)-(16) would entail, say, that everyone who knows just Gal is lucky, an entailment that the sentence clearly lacks (conversely, it would lead to meanings that are weaker than observed if this sentence were embedded under negation; see, e.g., Ruys 1992, Mayr and Spector 2012, Ruys and Spector 2017 on probing parses with stronger meanings).[3]

(15) Everyone who knows both Gal and Tal is lucky.
Cannot convey: (everyone who knows Gal is lucky) $\wedge$ (everyone who knows Tal is lucky)

(16) Everyone who knows every student is lucky.
Cannot convey: $\forall$x: student x $\rightarrow$ (everyone who knows student x is lucky)

The unavailability of exceptional scope of conjunction and universal quantifiers can be brought out more sharply with examples in which the low-scope, but not the exceptional scope, construal of the quantifier is pragmatically infelicitous. One such example is provided in (17) (due to Winter

[3]In order to avoid a parse of the sentence on which the conjunction of proper names would give rise to a homogeneity inference, and hence potentially obfuscate the difference between disjunction and conjunction, we employ *both* that is immediately adjacent to the conjunction and that blocks the homogeneity inference (cf., e.g., Szabolcsi and Haddican 2004, Schmitt 2013, 2019, Bar-Lev 2018). See fn. 11 as well as Sect. 4.1 for related discussion.

2002): if the universal quantifier could take exceptional scope, we would get a reasonable meaning that no matter who Gal's mother is, Gal should be fine. The fact that we only get an odd meaning for the sentence indicates that exceptional scope cannot be assigned to universal quantifiers.

(17) %If every woman gave birth to Gal, Gal should be fine.

On the basis of such, and many other arguments, we conclude that a low-scope construal of conjunction and universal quantifiers is the only one available in examples like the above ones – that is, conjunction and universal quantifers cannot take scope out of syntactic islands.

Exceptional movement. Various analyses of exceptional scope readings of disjunction and indefinites have been proposed (e.g., Reinhart 1997, Winter 1997, Kratzer 1998, Matthewson 1999, Schwarzschild 2002, Schlenker 2006, Endriss 2009, Brasoveanu and Farkas 2011, among many others). For concreteness, we assume that disjunction and indefinites have their standard quantificational meanings, and that exceptional scope is achieved by quantifier raising (e.g., Geurts 2000, Schwarz 2001, see also Barker 2022 for discussion). On this approach, the sentences in (13)-(14) have the LFs in (18), where the disjunctive and indefinite phrases have moved out of the relative clause.

(18) a. [Gal or Tal] [everyone who knows ~~[Gal or Tal]~~ is lucky]
b. [a student] [everyone who knows ~~[a student]~~ is lucky]

possible exceptional movement

Movement structures are interpreted by having the lower copies of the moved phrases transformed into definite descriptions of objects bound by the moved phrase (Trace Conversion, e.g., Fox 2002, Sauerland 2004a). The meaning of the structures in (18) are provided in (19) – they correspond to the target exceptional scope readings of the disjunction and the indefinite.

(19) a. (everyone who knows Gal is lucky) $\vee$ (everyone who knows Tal is lucky)
b. $\exists$x: student x $\rightarrow$ (everyone who knows student x is lucky)

Parallel LFs are unavailable for the corresponding sentences with conjunction and universal quantifiers, as indicated in (20). This holds by fiat on the approach here, but can be derived on principled grounds on more sophisticated movement approaches (esp., Demirok 2019, Ch. 4, on whose proposal anomalous truth conditions are assigned to structures in which non-indefinites take exceptional scope, and Charlow 2020, on whose proposal only nominals that are underlyingly predicative, such as indefinites, can undergo exceptional scope shift).

(20) a. *[Gal and Tal] [everyone who knows ~~[Gal and Tal]~~ is lucky]

b. *[every student] [everyone who knows ~~[every student]~~ is lucky]

impossible movement

While the simple movement theory of exceptional scope may ultimately prove to be too simplistic, we believe that its adoption is not crucial for the purposes of the paper, and that the convenience of its simplicity outweighs here its familiar shortcomings (though see Sect. 4.3). More to the point, any theory that (i) assigns exceptional scope to the existential quantification introduced by disjunction and indefinites, or something akin to it, and (ii) does not assign something similar to conjunction and universal quantifiers, would do for our purposes. In this respect, the choice function approaches with existential closure (e.g., Reinhart 1997, Winter 1997, Matthewson 1999) would do as well. To support this claim, we consider alternative approaches at different points below.

Missing and available alternatives. A consequence of the definition of alternatives in (5) is that sentences in which disjunction and indefinites take exceptional scope lack parallel alternatives in which these expressions are replaced by conjunction and universal quantifiers, as stated in (21) – namely, grammar cannot generate them (but see Charlow 2019 for an alternative assumption).

(21) a. *[Gal and Tal] [everyone who knows ~~[Gal and Tal]~~ is lucky]
$\notin$ ALT([Gal or Tal] [everyone who knows ~~[Gal or Tal]~~ is lucky])

b. *[every student] [everyone who knows ~~[every student]~~ is lucky]
$\notin$ ALT([a student] [everyone who knows ~~[a student]~~ is lucky])

This is not to say, however, that such sentences lack alternatives altogether. Which ones they induce is again constrained by the characterization in (5) – specifically, by limitations of structural complexity and by what the grammar is able to generate. Under our representation of movement, we expect the disjunctive sentence in (18a) to have the alternatives in (22): the low-scope disjunct alternatives (rows 1-2), the low-scope disjunctive alternative (row 3), and the low-scope conjunctive alternative (row 4). These alternatives are derived by deleting the high copy of the moved element at the LF, and appropriately substituting the low copy with its alternatives.

(22) [everyone who knows Gal is lucky],

[everyone who knows Tal is lucky],

[everyone who knows [Gal or Tal] is lucky],

[everyone who knows [Gal and Tal] is lucky]

$\in$ ALT([Gal or Tal] [everyone who knows ~~[Gal or Tal]~~ is lucky])

The indefinite sentence in (18b) has the alternatives provided in (23): it has the simple subdomain alternatives (row 2), the low-scope existential quantifier subdomain alternatives (row 3), and the low-scope universal quantifier subdomain alternatives (row 4). If Gal and Tal are the only relevant students, the subdomain alternatives correspond to the disjunct alternatives in (22), and the low-scope quantifier alternatives correspond to the low-scope disjunctive and conjunctive alternatives.

(23) For any D′, where $[\![D']\!] \subseteq [\![D]\!]$,

[a$_{D'}$ student] [everyone who knows ~~[a$_{D'}$ student]~~ is lucky],

[everyone who knows [a$_{D'}$ student] is lucky],

[everyone who knows [every$_{D'}$ student] is lucky]

$\in$ ALT([a$_D$ student] [everyone who knows ~~[a$_D$ student]~~ is lucky])

In short, we expect the surface forms of the initial sentences, on which disjunction and indefinites are interpreted in the relative clause, to be alternatives to the initial sentences. These alternatives will not play an important role in much of the paper, but we substantiate their assumption in the following by showing that (i) they correctly inhibit potential conjunctive strengthenings as well as that (ii) they allow us to derive the observed strengthened readings of the sentences we just discussed.

Inhibition. The absence of parallel conjunctive and universal quantifier alternatives may have considerable repercussions for what inferences disjunctive and indefinite sentences can give rise to. Missing a parallel conjunctive or universal quantifier alternative is not enough by itself, however, to warrant conjunctive strengthening. This is the case for the sentences in (13)-(14) discussed above – even though their exceptional scope structures lack parallel conjunctive and universal quantifier alternatives, their sets of alternatives are closed under conjunction. For example, this holds for the set of alternatives to the sentence (13), provided in (22) above: in particular, the conjunction of the disjunct alternatives is equivalent to the low-scope disjunctive alternative.[4] Accordingly, conjunctive strengthening of the sentence is predicted not to be possible.

(24) **Condition (C) precludes potential conjunctive inference for (13):**

ALT([Gal or Tal] [everyone who knows ~~[Gal or Tal]~~ is lucky]) is closed under conjunction.

$\big($esp. $[\![$everyone who know Gal is lucky$]\!] \wedge [\![$everyone who knows Tal is lucky$]\!]$

$\Leftrightarrow [\![$everyone who knows Gal or Tal is lucky$]\!]\big)$

In fact, on its exceptional scope parse, the sentence in (13) gives rise to the negation of the conjunctive inference, namely, that it is not the case that both Gal and Tal are such that everyone who knows them is lucky (as discussed by Charlow 2019 in his pioneering work on scalar implicatures of exceptional scope elements; see Sect. 5.1 for further discussion). This inference can be derived precisely by negating the low-scope disjunctive alternative.[5] The same reasoning and conclusions extend to the indefinite sentence in (14).[6] We can thus take our representations and the alternatives

[4]This equivalence holds on the assumption that either universal quantifiers do not induce presuppositions that their restrictors are non-empty (these would then have to arise as implicatures, say) or that these are factored out in the application of condition (C). With respect to the latter option, it is worth noting that a felicitous assertion of the exceptional scope disjunctive sentence in (13) requires the existential presuppositions of both disjuncts to be satisfied in the context (cf. Karttunen 1974). Now, in any such context, the conjunction of the disjunct alternatives and the low-scope disjunctive alternative are contextually equivalent. As will become clear in our actual computations of conjunctive strengthening, such contextual equivalence suffices for blocking conjunctive strengthening.

[5]The inference is derived by means of exhaustification, which we introduce in the following section. Following Fox 2007, we implement it by attaching an exhaustification operator *exh* at matrix level of the sentence, which then negates the low-scope disjunctive alternative in the case at hand (the low-scope disjunctive alternative is excludable, while the disjunct and the low-scope conjunctive alternatives are not). See Sect. 2.2 for definitions and details, and Sect. 5.1 for related discussion.

[6]If disjunction has more than two disjuncts, or if the domain of the indefinite in (14) consists of more than two students, multiple readings can be derived for the sentence. The readings vary with respect to which of the alternatives to the sentences are contextually relevant. Focusing on the indefinite sentence, for example, if no proper subdomain alternatives are relevant – that is, if only the low-scope existential quantifier alternatives with the same domain as the initial indefinite is relevant – the interpretation of (14) is that not every student is such that everyone who knows them is lucky. In contrast, if all alternatives are relevant, the reading that is derived for (14) is that exactly one student is such that everyone who knows them is lucky. Charlow 2019 notes that both readings are indeed attested. A reviewer further suggests that intermediate readings – readings that are entailed by the latter and entail the former – are expected to also arise in suitable contexts. Whether such readings are indeed available, and whether they can be derived by the theory as independent readings, are important questions that must be left for future work. Addressing them will require a parallel

they induce to be supported, both (i) by their blocking of undesirable inferences and (ii) by their enabling the derivation of observed inferences.[7]

Avoiding inhibition? We have thus not yet shown that exceptional scope asymmetries affect strengthening in ways conducive to generating conjunctive inferences. Are there cases in which low-scope alternatives to exceptional scope structures <u>do not inhibit</u> conjunctive strengthening? As we have just seen, identifying such cases requires isolating environments that satisfy three conditions. (i) The environment must not be anti-additive – that is, replacing disjunction in the environment with its disjuncts and then conjoing the resulting structures should not yield a meaning that is equivalent to that of the initial environment (since, in anti-additive environments, the hypothetical conjunctive inference would be equivalent to the low-scope disjunctive/existential quantifier alternative). (ii) The environment must not be distributive – that is, universal quantifiers being dominated by the environment should result in a different meaning than when they scope above the environment (since, in distributive environments, the hypothetical conjunctive inference would be equivalent to the low-scope conjunctive/universal quantifier alternative). (iii) If disjunction or an indefinite occur in an upward-monotone environment, conjunctive strengthening of exceptional scope elements might be confounded with conjunctive strengthening of the sentences on their surface form construal (see the discussion of sentences (1)-(2) above). One class of cases that satisfies these boundary conditions involves disjunction and indefinites appearing in the restrictor of proportional quantifiers.

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development of a theory of pruning of alternatives (cf. Crnič et al. 2015, Bar-Lev 2018). See also fn. 9.

[7] A reviewer notes that the sentence in (i), in which a universal quantifier is topicalized across a modified numeral indefinite in the subject position, conveys on neutral intonation that it is false that more than two female linguists read every book. This inference can be derived by negating the low-scope universal quantifier alternative to the sentence, provided in (iib), tentatively lending further support to our assumption of low-scope alternatives. A host of further predictions are generated on the assumption of low-scope alternatives, predictions that we cannot explore further here.

(i) Jedes Buch haben mehr als zwei Linguistinnen gelesen. [German]
every book have more than 2 linguists.F read
"Every book is such that more than two female linguists read it."

(ii) a. LF: [every book] [[more than 2 linguists] read ~~[every book]~~]
b. Alternative: [[more than 2 linguists] read [every book]]

2.2 Supporting evidence

Consider the sentence in (25), in which disjunction occurs in the restrictor of *most*, a non-monotone environment (see Bar-Lev and Fox 2020, fn. 46), as well as the sentence in (26), in which disjunction is replaced with its nominal cousin, an *either* phrase. (As suggested above, the sentences may have to be read with focal stress on the disjunctive marker and the indefinite determiner in order to obtain the target readings we discuss below.)

(25) Most kids that are on team A or team B are on both teams.

(26) Most kids that are on either team are on both teams.

Conjunctive and missing disjunctive meaning. The sentences in question <u>can</u> correctly describe a scenario involving two teams, team A and team B, each consisting of 5 kids, with 3 kids belonging to both teams (that is, 4 of the 7 kids are on a single team, 3 of the 7 kids are on both teams). Under this setup, the sentences would be false if disjunction and *either* contributed their standard meanings in their surface positions, as in (27a), since only 3 of the 7 kids are on both teams. However, the sentences are true if the conjunctive inference is computed, as in (27b), as indeed each team is such that the majority of kids on it that are also on the other team (both times 3 of the 5 kids). (Although we have replicated these judgments for translations of (25)-(26), as well as of subsequent examples, in several Indo-European and Semitic languages, we limit our discussion to English for reasons of brevity; see also fn. 10 for related discussion.)

(27) **Possible reading of sentences (25)-(26):**

a. $\not\Rightarrow$ $[\![\text{most}]\!](\{x \mid \text{kid x on team A or B}\})(\{x \mid \text{kid x on both teams}\})$

b. $\Rightarrow$ $[\![\text{most}]\!](\{x \mid \text{kid x on team A}\})(\{x \mid \text{kid x on both teams}\}) \wedge$
$[\![\text{most}]\!](\{x \mid \text{kid x on team B}\})(\{x \mid \text{kid x on both teams}\})$

That the sentences in (25)-(26) can convey the conjunctive inference in (27b) – rather than merely an exceptional scope disjunctive one – is intuitively obvious. For example, if one accepts the sentences in (25)-(26), it is natural to respond, when asked about either team, that most of its members are on both teams. Relatedly, these sentences can be felicitously followed by a clarification question whether the speaker indeed meant to convey that most kids on team A are also on team B (cf. Meyer 2013 on this diagnostic for optional inferences). Finally, disjunctions that are not conjunctively strengthened typically give rise to ignorance inferences regarding which of the disjuncts holds – yet such inferences are clearly absent here (see, e.g., Sauerland 2004b, Fox 2007, Meyer 2013 for a discussion of the apparent obligatoriness of ignorance inferences).[8] (See Sect. 5.2 for a further discussion of the readings of such sentences on their exceptional scope construal.)

In the following, our discussion tracks the entailment patterns described in (27). The first pattern pertains to the scope of disjunction and indefinites: they do not contribute their meaning in their surface positions, but at the matrix level. However, what we observe at the matrix level are conjunctive meanings, not merely disjunctive or existential ones. This is captured by exhaustification.

Scope. Disjunction and the *either* NP can evade contributing their meaning in their surface scope positions by taking exceptional scope. Specifically, the sentences in (25)-(26) can be assigned the exceptional scope structures in (28), which have the meaning in (29).

(28) a. [team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams]
b. [either team] [most kids that are on ~~[either team]~~ are on both teams]

(29) $[\![\text{most}]\!](\{x \mid \text{kid } x \text{ is on team A}\})(\{x \mid \text{kid } x \text{ is on team A and team B}\}) \vee$
$[\![\text{most}]\!](\{x \mid \text{kid } x \text{ is on team B}\})(\{x \mid \text{kid } x \text{ is on team A and team B}\})$

The meaning in (29) is clearly independent of the low-scope disjunctive one – for example, it may be that most kids on team A are on both teams, but constitute only a small minority on team B. Nonetheless, the exceptional scope disjunctive meaning in (29) does not exhaust the meaning of (25)-(26); it is not strong enough, as these sentences give rise to a conjunctive inference. In what follows, we focus on the disjunctive sentence – the conjunctive strengthening of the corresponding indefinite sentence proceeds in an analogous fashion.

Strengthening. The structure in (28a) lacks a parallel conjunctive quantifier alternative since this is not generated by grammar, as stated in (30). It has the alternatives in (31): the initial sentence, a

[8]Similar judgments are available for sentences with other proportional quantifiers, such as those provided in (i), in apropriately modified scenarios. Our account of the pattern in the main text extends to these data in obvious ways:

(i) a. More than a third of the kids that are on team A or team B are on both teams.
b. Exactly one third of the kids that are on team A or team B are on both teams.
c. 60% of the kids that are on team A or team B are on both teams.

low-scope disjunctive alternative, two disjunct alternatives, and a low-scope conjunctive alternative.

(30) *[team A and team B] [most kids that are on ~~[team A and team B]~~ are on both teams]

$\not\in$ ALT([team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams])

(31) ALT([team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams]) =

{ [team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams] ,
[most kids that are on [team A or team B] are on both teams],
[most kids that are on team A are on both teams],
[most kids that are on team B are on both teams],
[most kids that are on [team A and team B] are on both teams] }

The set of alternatives in (31) is not closed under conjunction: in particular, the conjunction of the disjunct alternatives is not equivalent to the low-scope disjunctive alternatives (as witnessed by our initial scenario on which the former is true but the latter is false) or any other alternative for that matter. Accordingly, conjunctive strengthening may be possible.

(32) **Condition (C) admits potential conjunctive inference for (25):**

ALT([team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams])

is not closed under conjunction.

Conjunctive strengthening is effected by exhaustification, which is encoded in operator *exh*. *Exh* can attach at any clausal level, and it negates all relevant excludable alternatives, as defined in (33)-(34) (the definition is from Katzir 2014, Crnič et al. 2015, a slight revision of Fox 2007). We represent excludable alternatives as semantic objects, as interpretations of elements that are in all maximal subsets of formal alternatives to a sentence whose joint negation is consistent with the sentence.

(33) **Exhaustification:**

$[\![\text{exh}_C \text{ S}]\!] = [\![\text{S}]\!] \wedge \forall \text{p} \in \text{IE(S)} \cap \text{C}: \neg \text{p}$

(34) **Excludable alternatives:**

IE(S) = $\bigcap\{$M$'$ | for a maximal subset M of ALT(S) such that $\{\neg[\![\text{S}']\!] \mid \text{S}' \in \text{M}\} \cup \{[\![\text{S}]\!]\}$ is consistent, M$'$ is the set of meanings of the elements of M$\}$

With these definitions in hand, we can turn to the sentence in (25). The full parse of the sentence that yields the conjunctive inference involves recursive exhaustification, as provided in (35):

(35) [exh$_{C'}$ [exh$_C$ [teamA or teamB] [most kids that are on ~~[teamA or teamB]~~ are on both teams]]]

At the first layer of exhaustification, the low-scope disjunctive alternative is excludable, as provided in (36). This is because it may be true that either most students on team A or most kids on team B are on both teams, while it is false that most of all the kids are on both teams (cf. the scenario discussed above, in which the majority of team A is also on team B, but team B is larger, hence team A kids make up only a minority of its members). Since the conjunctive alternative is a tautology, and since the disjunct alternatives are symmetric, none of these alternatives are excludable.

(36) IE([team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams]) =

$$\Big\{ [\![\text{most}]\!](\{\text{x} \mid \text{kid x is on team A or B}\})(\{\text{x} \mid \text{kid x is on team A and team B}\}) \Big\}$$

The output of the first layer of exhaustification on the assumption that the low-scope disjunctive alternative is not relevant is provided in (37) (we assume throughout that non-disjunct alternatives are not relevant in order to simplify our derivations[9]): it corresponds to the meaning of the sister of *exh*, that most kids on team A or most kids on team B are on both teams.

(37) $[\![$[exh$_C$ [teamA or teamB] [most kids that are on ~~[teamA or teamB]~~ are on both teams]]$]\!]$ =

$$\big([\![\text{most}]\!](\{\text{x} \mid \text{kid x is on team A}\})(\{\text{x} \mid \text{kid x is on team A and team B}\}) \vee$$
$$[\![\text{most}]\!](\{\text{x} \mid \text{kid x is on team B}\})(\{\text{x} \mid \text{kid x is on team A and team B}\})\big)$$

At the second layer of exhaustification, the disjunct alternatives become excludable, as provided in (38), this is evidenced by their joint exclusion being compatible with the sentence being true. The output of the matrix exhaustification on our standing assumption that only disjunct alternatives are relevant is given in (39). Specifically, exhaustification yields, for each team, the inference that if most of its members are on both teams, then the same must hold for the other team. Since this condition

[9]In many cases, if additional alternatives are taken to be relevant (= not pruned), additional inferences may be generated. In the case at hand, the additional inference would be that it is false that most of all the kids are on both teams. In the one case below in which pruning is necessary to generate a conjunctive inference, this fact is discussed explicitly (see fn. 20). Finally, where (exhaustified) disjunct alternatives are excludable, their pruning is constrained by a principle that requires their joint resolution (cf. Bar-Lev and Fox 2020 on 'cell identification'). See Gotzner and Romoli 2022 for a review of data that suggest ease of pruning of non-disjunct alternatives, in contrast to disjunct alternatives.

holds of one team, as established in (37), it must hold of both teams.

(38) IE([exh$_C$ [teamA or teamB] [most kids that are on ~~[teamA or teamB]~~ are on both teams]]) =

$$\{\ [\![\text{most}]\!](\{x \mid \text{kid x is on team A}\})(\{x \mid \text{kid x is on team A and team B}\}) \wedge$$
$$\neg[\![\text{most}]\!](\{x \mid \text{kid x is on team B}\})(\{x \mid \text{kid x is on team A and team B}\}),$$
$$[\![\text{most}]\!](\{x \mid \text{kid x on team B}\})(\{x \mid \text{kid x is on team A and team B}\}) \wedge$$
$$\neg[\![\text{most}]\!](\{x \mid \text{kid x is on team A}\})(\{x \mid \text{kid x is on team A and team B}\}),$$
$$[\![\text{most}]\!](\{x \mid \text{kid x is on team A or B}\})(\{x \mid \text{kid x is on team A and team B}\}) \wedge$$
$$\neg[\![\text{most}]\!](\{x \mid \text{kid x is on team A}\})(\{x \mid \text{kid x is on team A and team B}\}) \wedge$$
$$\neg[\![\text{most}]\!](\{x \mid \text{kid x is on team B}\})(\{x \mid \text{kid x is on team A and team B}\})\ \}$$

(39) $[\![$[exh$_{C'}$ [exh$_C$ [tmA or tmB] [most kids that are on ~~[tmA or tmB]~~ are on both teams]]]$]\!]$ =

$$[\![\text{most}]\!](\{x \mid \text{kid x is on team A}\})(\{x \mid \text{kid x is on team A and team B}\}) \wedge$$
$$[\![\text{most}]\!](\{x \mid \text{kid x is on team B}\})(\{x \mid \text{kid x is on team A and team B}\})$$

The meaning in (39) corresponds to the conjunctive inference that most kids on team A are on both teams, and most kids on team B are on both teams. We thus showed that this inference follows naturally on the assumptions set out above. First: The lack of a low-scope disjunctive contribution of disjunction follows from it being assigned exceptional scope. In fact, the low-scope disjunctive meaning can be negated by exhaustification. Second: The sentence on this exceptional scope parse satisfies condition (C), and exhaustification strengthens its meaning to a conjunctive one.[10]

2.3 Excursus: Choice functions and alternatives

We conjectured that our choice of a particular theory of exceptional scope was not crucial to our analysis. We provide some support for this claim by discussing a prominent alternative account, according to which disjunction and indefinites introduce choice functions that are existentially closed,

[10]It is worth noting that the parse that gives rise to the target inference in the main text is more complex than those that do not (the former involves, at least, one additional application of scope shift). Accordingly, it may well be less accessible and preferred than other parses of the sentence (cf., e.g., Fodor and Garrett 1966, Fodor et al. 1974, Frazier and Rayner 1982). Although the speakers that we consulted about the data accessed the target reading on appropriate intonation, here and for the other examples in the paper, other disambiguations were often initially preferred. Moreover, some of the speakers accessed the target readings more easily with indefinite sentences. The same holds for the sentences in other languages whose speakers we consulted (Slovenian, German, Arabic, Hebrew).

as defined in (40) (e.g., Reinhart 1997, Winter 1997, 2002, Matthewson 1999, Schlenker 2006).

(40) **Choice functions and existential closure:**
Let E be a non-empty set of individuals. A function f: $\mathscr{P}$(E) $\rightarrow$ E is a choice function, f $\in$ CH, iff for every A $\subseteq$ E: if A is not empty then f(A) $\in$ A. Choice function variables are bound by an existential closure operator $\exists$ that can attach at any clausal level.

Sentences (13)-(14) may be assigned the structures in (41a)-(42a): matrix existential closure binds the choice function variables introduced by disjunction and the indefinite. The choice functions' arguments are, respectively, the set of individuals denoted by the disjoined expressions and the domain of the indefinite. The resulting interpretations, shown in (41b)-(42b), correspond to the exceptional scope readings of disjunction and the indefinite.

(41) a. [$\exists_f$ [everyone who knows [Gal or$_f$ Tal] is lucky]
b. $\exists$f$\in$CH: (everyone who knows f({Gal, Tal}) is lucky)
$\Leftrightarrow$ (everyone who knows Gal is lucky) $\vee$ (everyone who knows Tal is lucky)

(42) a. [$\exists_f$ [everyone who knows [a$_f$ student] is lucky]
b. $\exists$f$\in$CH: (everyone who knows f({x | x student}) is lucky)
$\Leftrightarrow$ $\exists$x: student x $\wedge$ (everyone who knows student x is lucky)

In line with our above assumptions, the structures in (41)-(42) lack parallel conjunctive and universal quantifier alternatives since grammar cannot generate them, either due to an independent constraint on their movement or the lexicon not containing a universal closure operator (but see Charlow 2019 for an alternative assumption):

(43) *[Gal and Tal] [everyone who knows ~~[Gal and Tal]~~ is lucky],
*[$\forall_f$ [everyone who knows [Gal or$_f$ Tal] is lucky]]
$\notin$ ALT([$\exists_f$ [everyone who knows [Gal or$_f$ Tal] is lucky])

(44) *[every student] [everyone who knows ~~[every student]~~ is lucky],
*[$\forall_f$ [everyone who knows [a$_f$ student] is lucky]]
$\notin$ ALT([$\exists_f$ [everyone who knows [a$_f$ student] is lucky])

The choice function representations admit low-scope connective and quantifier alternatives. The low-scope conjunctive and universal quantifier alternatives are admitted since the existential closure operator can be removed and the conjunctive marker replace the disjunctive one, as in (45), or the universal quantifier replace the indefinite, as in (46).

(45) [everyone who knows [Gal and Tal] is lucky]

$\in$ ALT([$\exists_f$ [everyone who knows [Gal or$_f$ Tal] is lucky])

(46) [everyone who knows [every student] is lucky]

$\in$ ALT([$\exists_f$ [everyone who knows [a$_f$ student] is lucky])

The structures in (41a)-(42a) plausibly have low-scope disjunctive or existential alternatives as well, as stated in (47)-(48). The source of these alternatives depends on the details of the theory, however. It may either follow from disjunction and indefinites being ambiguous and allowing for both standard and choice function construals (cf., e.g., Reinhart 1997, Kratzer 1998, Matthewson 1999, but not Winter 1997, 2002), or by taking existential closure to (potentially vacuously) apply at each clausal level, as suggested by a reviewer (or by taking existential closure to apply in tandem with compositional interpretation).

(47) [everyone [who knows [Gal or Tal]] is lucky], or
[everyone [$\exists_f$ who knows [Gal or$_f$ Tal]] is lucky]

$\in$ ALT([$\exists_f$ [everyone who knows [f Gal or Tal] is lucky])

(48) [everyone [who knows [a student]] is lucky], or
[everyone [$\exists_f$ who knows [a$_f$ student]] is lucky]

$\in$ ALT([$\exists_f$ [everyone who knows [a$_f$ student] is lucky])

The choice function approach to disjunction and indefinites thus does not seem to differ in any relevant respect from the simple movement approach, at least for the purposes of the present discussion. Both involve structures in which disjunction and indefinites, or correspondingly existential quantification over disjuncts, apply at the level of exceptional scope. Both allow for the generation of subdomain alternatives (existential closure over choice functions can be restricted). Both predict that there are no exceptional scope conjunction and universal quantifier alternatives. And both admit low-scope connective and quantifier alternatives. In the interest of simplicity, we stick to the simple movement approach in what follows (but see Sect. 4.3 for further discussion).

3 Another asymmetry in scope taking

Another difference in the behavior of disjunction and indefinites vs. conjunction and universal quantifiers is that the former, but not the latter, can take scope out of certain downward-monotone environments. Consequently, sentences in which this happens lack parallel conjunctive and universal quantifier alternatives. Their sets of alternatives might thus not be closed under conjunction, potentially giving rise to conjunctive inferences. We argue that this possibility, too, is realized.

3.1 Scoping out of downward-monotone environments

Disjunction and indefinites can scope out of certain downward-monotone environments, whereas conjunction and universal quantifiers cannot (see, e.g., Mayr and Spector 2012, Fleisher 2015; see also Liu 1990, Beghelli 1995 for related earlier discussion). For example, disjunction and indefinite may move covertly above downward-monotone operators like *fewer than 1000 students* and Strawson downward-monotone operators like *only 900 students* (that is, operators that are downward-monotone once their presuppositions are factored out, von Fintel 1999). This is exemplified in (49)-(50), where the wide-scope readings of disjunction and the indefinite are forced by the continuation "I don't remember which."

(49) a. Fewer than 1000 students got into Cambridge or Oxford. I don't remember which.
     b. Fewer than 1000 students got into a UK university. I don't remember which.
        Reading: ✓ or/a > fewer than 1000

(50) a. Only 900 students got into Cambridge or Oxford. I don't remember which.
     b. Only 900 students got into a UK university. I don't remember which.
        Reading: ✓ or/a > only 900

This scopal behavior is not replicated with conjunction and universal quantifiers, as exemplified in (51)-(52) (esp., Fleisher 2015, fn.25, for the behavior of universal quantifiers under modified numeral indefinites). The sentences in (51)-(52) thus do not have parses on which conjunction and the universal quantifier would move covertly over the (Strawson) downward-monotone operators (see fn. 3 above on the use of *both* in these examples).[11]

[11] A reviewer inquired about the readings of a variant of (51) in which *each* is replaced by *every*. While the literature has primarily focused on *every* (perhaps because *each* NPs seem to be marked in certain downward-monotone environments, cf. Beghelli and Stowell 1997), it may well be that the scopal behavior of *each* is more flexible, at least for

(51) a. Fewer than 1000 students got into both Cambridge and Oxford.

b. Fewer than 1000 students got into every UK university.

Readings: ✓ fewer > and/every; ✗ and/every > fewer

(52) a. Only 900 students got into both Cambridge or Oxford.

b. Only 900 students got into every UK university.

Readings: ✓ only > and/every; ✗ and/every > only

A comprehensive description of this behavior as well as its explanation is still outstanding. One promising approach takes the unavailability of wide-scope for conjuntion and universal quantifiers in these configurations to follow from a condition on quantifier raising that requires it to weaken the meanings of the pertinent structures (see Mayr and Spector 2012 and Fleisher 2015, building on Fox 2000). What matters here is not the specific source of the restriction on scope, but rather the fact that certain disjunctive and indefinite sentences on the wide-scope construal of disjunction and indefinites lack parallel conjunctive and universal quantifier alternatives, as stated in (53)-(54) – again, this absence follows from grammar not generating such alternatives.

(53) *[Cambridge and Oxford] [fewer than 1000 students got into ~~[Cambridge and Oxford]~~]
$\notin$ ALT([Cambridge or Oxford] [fewer than 1000 students got into ~~[Cambridge or Oxford]~~])

(54) *[every UK university] [fewer than 1000 students got into ~~[every UK university]~~]
$\notin$ ALT([a UK university] [fewer than 1000 students got into ~~[a UK university]~~])

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some speakers. If this is indeed the case, and *each* can take wider scope, then, all else equal, such speakers should not compute conjunctive inferences for the examples at hand. However, if speakers who allow this greater scopal flexibility – including, it seems, the reviewer – nonetheless compute conjunctive inferences for these examples, the failure of the *each*-alternative to inhibit these inferences poses a challenge for the analysis offered in the main text, again assuming all else is equal. That said, not all else need be equal. Specifically, potential non-inhibition could stem from *each* not being an alternative to the indefinite determiner (as the reviewer notes). This possibility aligns with the observation that *each* behaves more like a high distributive head than a standard determiner (e.g., Sportiche 1988, Beghelli 1995, Beghelli and Stowell 1997, among others).

The reviewer also raises a related question about (complex) conjunctions. In this case, too, the same dialectic applies: to the extent speakers admit a wide-scope construal for certain conjunctive phrases in the examples at hand, yet still compute conjunctive inferences for the disjunctive sentences, those conjunctive phrases should, on our proposal, not count as alternatives (see, e.g., Dočekal et al. 2022 on conjunctive phrase structures of differing complexity). Given the scope and complexity of these issues, we have to leave their systematic study to future work.

As we will see, this absence of conjunctive and universal quantifier alternatives is consequential.

3.2 Supporting evidence

Consider the sentence in (55a), in which disjunction surfaces in the scope of a downward-monotone quantifier *fewer than 1000 students*, as well as the sentence in (55b), in which disjunction is replaced by its nominal cousin, an *either* phrase. Now, let's assume, more than 1000 students get into Oxford or Cambridge every year, though Oxford and Cambridge each accepts fewer than 1000 students. This state of affairs can be described by the sentences in (55). (Again, the disjunctive marker and the indefinite determiner may need to be stressed to access this reading.)

(55) a. Fewer than 1000 students got into Cambridge or Oxford.

b. Fewer than 1000 students got into either university.

This is unexpected on the surface scope construal of the sentence, whose meaning is provided in (56a), since it is false in the above scenario – namely, more than 1000 students got into one of these universities. Moreover, the sentences give rise to the conjunctive inference provided in (56b), so simple wide-scope disjunctive and indefinite construals of the sentences do not suffice to capture the sentences' meanings on their own.

(56) **Possible reading of sentences in (55):**

a. $\not\Rightarrow$ card({x | student x got into Cambridge or Oxford})$<$1000

b. $\Rightarrow$ card({x | student x got into Cambridge})$<$1000 $\wedge$
card({x | student x got into Oxford})$<$1000

A similar state of affairs obtains also with occurrences of disjunction and the *either* phrase in the scope of Strawson downward-monotone operators like the *only* NP in (57). We can judge the sentences as good descriptions of the scenario in which exactly 900 students got into Cambridge and exactly 900 other students got into Oxford.

(57) a. Only 900 students got into Cambridge or Oxford.

b. Only 900 students got into either university.

The fact that we can judge the sentences as true in the above scenario indicates that they do not convey a low-scope disjunctive or existential meaning, provided in (58a), as more than 900 students got into Cambridge or Oxford. They also gives rise to the conjunctive inference in (58b).

(58) **Possible reading of sentences in (57):**

a. $\not\Rightarrow$ card({x | student x got into Cambridge or Oxford}) = 900

b. $\Rightarrow$ card({x | student x got into Cambridge}) = 900 $\wedge$
card({x | student x got into Oxford}) = 900

These patterns match those discussed in the preceding section: disjunction and indefinites do not contribute a disjunctive/existential meaning in the environment in which they appear at surface form, rather they convey a wide-scope conjunctive meaning. Accordingly, the derivation of these patterns can proceed in parallel to those in the preceding section.

Scope and strengthening. We focus on the disjunctive sentence in (55) in the following, with the derivation being straightforwardly extendable to the indefinite sentence in (57). The sentence can be parsed with disjunction taking matrix scope, as provided in (59). But the wide-scope disjunctive meaning, provided in (60), is on its own weaker than the reading we have to derive for the sentence.

(59) [Cambridge or Oxford] [fewer than 1000 students got into ~~[Cambridge or Oxford]~~]

(60) (card({x | student x got into Cam.}) = 900) $\vee$ (card({x | student x got into Oxford}) = 900)

The meaning of the structure in (59) can be strengthened to a conjunctive one. The set of alternatives to the structure is provided in (61), which crucially lacks a wide-scope conjunctive alternative.

(61) ALT([Cambridge or Oxford] [fewer than 1000 students got into ~~[Cambridge or Oxford]~~]) =
{ [Cambridge or Oxford] [fewer than 1000 students got into ~~[Cambridge or Oxford]~~],
[fewer than 1000 students got into [Cambridge or Oxford]],
[fewer than 1000 students got into Cambridge],
[fewer than 1000 students got into Oxford],
[fewer than 1000 students got into [Cambridge and Oxford]] }

The set in (61) is not closed under conjunction, as stated in (62). In particular, the conjunction of the disjunct alternatives is not equivalent to the low-scope disjunctive alternative, as witnessed by the conjunction of the disjunct alternatives being true in the above scenario and the low-scope disjunctive alternative being false. Hence, conjunctive strengthening of the sentence may be possible.

(62) **Condition (C) admits potential conjunctive inference for (55):**

ALT([Cam or Oxford] [fewer than 1000 students got into ~~[Cam or Oxford]~~]])

is not closed under conjunction.

At the first layer of exhaustification, the only excludable alternative to the base sentence is the low-scope disjunctive one. At the second layer of exhaustification, all the alternatives, including the disjunct alternatives, are excludable, as provided in (63). (Recall that we are treating only the disjunct alternatives as relevant, that is, in the domains C and C'.)

(63) IE([exh_C [Cam. or Oxford] [fewer than 1000 students got into ~~Cam. or Oxford~~]]) =

$$\begin{aligned}\{ &\text{card}(\{x \mid \text{student x got into Cambridge}\}) < 1000 \wedge \\ &\quad \neg(\text{card}(\{x \mid \text{student x got into Oxford}\}) < 1000), \\ &\text{card}(\{x \mid \text{student x got into Oxford}\}) < 1000 \wedge \\ &\quad \neg(\text{card}(\{x \mid \text{student x got into Cambridge}\}) < 1000), \\ &\text{card}(\{x \mid \text{student x got into Cambridge or Oxford}\}) < 1000, \\ &\text{card}(\{x \mid \text{student x got into Cambridge and Oxford}\}) < 1000 \wedge \\ &\quad \neg(\text{card}(\{x \mid \text{student x got into Oxford}\}) < 1000) \wedge \\ &\qquad \neg(\text{card}(\{x \mid \text{student x got into Oxford}\}) < 1000) \}\end{aligned}$$

The recursively exhaustified meaning of the sentence is provided in (64): in particular, negating the disjunct alternatives gives us the proposition that one disjunct alternative is true iff the other one is true as well, and since the sentence entails that at least one of them is true, both must be. Accordingly, the meaning we derived corresponds to the conjunctive inference of the sentence.

(64) ⟦[$\text{exh}_{C'}$ [exh_C [Cam. or Oxford] [fewer than 1000 students got into ~~[Cam. or Oxford]~~]]]]⟧ =

$$\begin{aligned}&\text{card}(\{x \mid \text{student x got into Cambridge}\}) < 1000 \wedge \\ &\quad \text{card}(\{x \mid \text{student x got into Oxford}\}) < 1000\end{aligned}$$

In conclusion, we have shown that scope and strengthening can conspire to generate conjunctive inferences also when disjunction and indefinites occur at surface form in downward-monotone environments. This is possible because, when disjunction and indefinites take scope out of such environments, the resulting sentences lack parallel conjunctive and universal quantifier alternatives, and the conjunctions of their disjunct alternatives are not equivalent to any other available alternatives.

3.3 Scope and strengthening in NPI licensing

Indefinites that we used to illustrate conjunctive strengthening were *either* NPs. These have a narrower distribution than regular indefinites. For example, they are unacceptable in simple episodic sentences, while they are acceptable in downward-monotone and modal environments.

(65) #Gal got into either university.

(66) a. Gal didn't get into either university.
b. Gal may/must have got into either university.

Moreover, in appropriately modified scenarios, *either* NPs can be replaced in all the above examples with *any* NPs without affecting the acceptability of the sentences or the kinds of inferences the sentences give rise to.[12] For example, consider the scenario in which each Ivy League school accepts fewer than 1000 students, but they jointly accept more than 1000 students. This state of affairs can be felicitously described with the sentence in (67): in short, the NPI does not contribute a low-scope existential meaning, but does give rise to a conjunctive inference, as stated in (68).

(67) Fewer than 1000 students got into any Ivy League school.

(68) **Possible reading of sentence (67):**
a. $\not\Rightarrow$ card({x | student x got into an Ivy Leauge school}) $<$ 1000
b. $\Rightarrow \bigwedge_{x \in [\![\text{Ivy Leaguge school}]\!]}$ card({x | student x got into x}) $<$ 1000

[12] Although *any* NPs and *either* NPs have similar distributions, these are not quite the same. In particular, *either* NPs, but not *any* NPs, are acceptable in universal modal sentences, as exemplified in (i). Accordingly, the two cannot be subject to the same licensing condition. Rather, *either* NPs could be classified as 'existential free choice items' (Chierchia 2013), and their distribution might be governed by exhaustification alone, while that of *any* NPs might be governed by silent *even* (cf. Crnič 2022).

(i) a. Gal must have got into either university.
b. *Gal must have got into any university.

The derivation of the meaning of the sentence in (67) proceeds in the same way as it did with disjunction and *either* NPs: the *any* NP takes exceptional scope, as in (69), and the sentence is recursively exhaustified to yield the conjunctive inference, provided in (70), where we continue to assume that only the existential subdomain alternatives are relevant for simplicity.

(69) [exh$_{C'}$ [exh$_{C}$ [any$_{D}$ IL school] [fewer than 1000 students got into ~~[any$_{D}$ IL school]~~]]]

(70) $[\![$[exh$_{C'}$ [exh$_{C}$ [any$_{D}$ IL school] [fewer than 1000 students got into ~~[any$_{D}$ IL school]~~]]]$]\!]$ =

$$\bigwedge_{x \in [\![\text{Ivy Leaguge school}]\!]} \text{card}(\{\text{x} \mid \text{student x got into x}\}) < 1000$$

So far so good. But how does such an analysis square with the fact that *any* NPs are NPIs? And how does it square with the fact that *either* NPs have a similarly restricted distribution? Perhaps surprisingly, the analysis is not only compatible with these expressions being NPIs, it bolsters support for a specific approach to NPI licensing, one that takes NPI licensing to be environment-based.

NPI Licensing. NPIs like *any* NPs have a famously restricted distribution.[13] Two approaches to the NPI licensing condition can be distinguished: an operator-based approach and an environment-based approach. According to the operator-based approach, originating with Fauconnier 1975 and Ladusaw 1979, *any* NPs are licensed only when they occur within the scope of a downward-monotone operator. This condition is not satisfied in the above examples, as the exhaustification operator *exh* is non-monotone, and no other operators c-command the NPIs. As such, the examples pose a further challenge to the operator-based formulation of the NPI licensing condition.

A reformulation of the NPI licensing condition in terms of environments, however, straightforwardly accounts for the behavior of NPI indefinites in the examples above, as well as for their broader distribution. The condition is stated in (71) (cf. Kadmon and Landman 1993, among others). (Since the licensing condition on *either* NPs is more complex, see fn. 12, and may not admit a fully informative formulation, we restrict our discussion to *any* NPs. Relatedly, see Crnič 2022 for an account of why certain NPIs, such as *ever*, resist conjunctive strengthening; the same reasoning as discussed in that paper applies in the present case.)

[13] We assume that *either* NPs and *any* NPs are not ambiguous between indefinites (sometimes called their 'NPI occurrences') and universal quantifiers (sometimes called their 'free choice item' occurrences). See, e.g., Chierchia 2004, 2013, Dayal 2013, Crnič 2017, 2019, 2022 for extensive discussion and arguments for *any* NPs and, e.g., Kratzer and Shimoyama 2002, Menéndez-Benito 2010, Fălăuş 2014, Alonso-Ovalle and Menéndez-Benito 2020, Fălăuş and Nicolae 2022 for analyses of 'free choice items' as indefinites in several other languages.

(71) An occurrence of *any* NP is acceptable iff it is dominated by a constituent that is (Strawson) downward-monotone with respect to its domain.

The condition is satisfied in all the examples discussed above, in which NPIs take exceptional scope and convey conjunctive meanings after exhaustification (see Crnič 2019, 2022 for an analogous discussion of more standard 'free choice occurrences' of *any* NPs).

Demonstration. Consider again the LF of sentence (67), provided in (69), and its interpretation in (70). In this LF, the NPI *any Ivy League school* scopes above a downward-monotone operator. If the meaning computed in (70) holds in a given scenario, then replacing the domain of the NPI with any subdomain results in a meaning that also holds in that scenario, as stated in (72) – at least on the assumption that NPIs induce only subdomain and scalar alternatives (see, e.g., Krifka 1995, Chierchia 2013 for the proposal, and Crnič 2019, 2022 for a discussion of its necessity). That is, in variants of the structure (69) in which *any* ranges over a smaller subdomain, we lose entailments pertaining to some of the elements in the initial domain of *any* – specifically, we lose some of the conjuncts in (70). The condition in (71) is accordingly satisfied.

(72) For any non-empty D $\subseteq$ [[Ivy League school]],

$$(\bigwedge_{x \in [\![\text{Ivy Leaguge school}]\!]} \text{card}(\{\text{x} \mid \text{student x got into x}\}) < 1000)$$
$$\Rightarrow (\bigwedge_{x \in D} \text{card}(\{\text{x} \mid \text{student x got into x}\}) < 1000)$$

The analysis of NPIs in these examples bears some resemblance to the now-deprecated universal quantifier analysis of NPIs (e.g., Quine 1960, Lasnik 1972), which treated *any* NPs as universal quantifiers that must scope above appropriate expressions. Crucially, however, our analysis diverges from these earlier accounts in that it makes no special assumptions about the nature of these expressions beyond two uncontroversial ones: namely, that they are indefinites and that they are subject to a licensing condition like (71). Moreover, our treatment above constitutes only one possible parse of the sentences (indeed, a dispreferred one), and is applicable only in environments in which exhaustification leads to conjunctive strengthening.[14,15]

[14]In relation to the analysis being applicable only in specific cases, consider for example the sentence in (i).

(i) Everyone who knows any girl is lucky.

If exceptional scope is assigned to the NPI in this sentence, it cannot be strengthened to convey a conjunctive inference (see Sect. 2.1 for disscusion). Accordingly, the parse in (iia) violates the condition in (71) – the contribution of the NPI matches that of other indefinites in upward-monotone environments, and so the sentence is not downward-monotone with respect to its domain on this parse. The sentence hence only has the surface scope parse in (iib).

(ii) a. #[any girl] [everyone who knows ~~[any girl]~~ is lucky]
b. [everyone who knows [any girl] is lucky]

Furthermore, if exceptional scope of an *any* NP is licensed – that is, if the sentence can undergo conjunctive strengthening – its scope cannot be probed with a sluicing continuation. This is illustrated in (iii).

(iii) #Fewer than 1000 students got into any Ivy League school. But I don't know which one.

The infelicity arises because the uniqueness presupposition induced by the question cannot be satisfied in such cases. The same effect is observed with all other expressions that give rise to universal inferences and that famously cannot feature as inner antecedents in sluicing configurations, as exemplified in (iv) (e.g., Chung et al. 1995).

(iv) #Gal got into every Ivy League school. But I don't know which one.

[15]We have observed that NPI indefinites can escape islands in Sect. 2.2. If they take matrix scope, they are licensed as discussed in the main text. But they can also escape islands without taking matrix scope. This can be appreciated by looking at a variant of Reinhart's 1997 well-worn examples in (i) (see also, e.g., Farkas 1981, Abusch 1993):

(i) a. [Context: A math textbook contains 500 difficult problems. Every math grad student is required to pick a problem and study every analysis that solves it. Tali studied every analysis that solves the four-color theorem. Zali studied every analysis that solves the Poincaré conjecture. But, as always, Gal is an exception.]
b. Gal DIDN'T study every analysis that solves ANY problem mentioned in the book.

The sentence has the plausible meaning that there is no problem in the book such that Gal studied its every analysis. This meaning can be derived as in (ii). Note that no exhaustification is required in this configuration in order to satisfy

4 Some predictions

The theory gives rise to a wide range of predictions. Here, we focus on those related to the trapping of the scope of disjunction and indefinites. Specifically, if the scope of these expressions can be successfully limited to their surface positions, the conjunctive inferences that we discussed above should no longer be available – at least not without the expressions also contributing their conventional meanings in their surface positions.

4.1 Disjunctive *either*

Larson 1985 showed that the scope of disjunction can be trapped by an appropriately placed *either* that associates with disjunction. In setting the stage, he observed that the sentence in (73), where *either* is adjacent to the disjunction, allows for three scope disambiguations:

(73) Sherlock pretended to look for either a burglar or a thief.
Readings: ✓ pretend>look for>or, ✓ pretend>or>look for, ✓ or>pretend>look for

In contrast, the sentence in (74), in which *either* is separated from the disjunction by a verb and a preposition, allows only for one disambiguation, the disambiguation on which the disjunction takes scope at the level at which *either* occurs at surface form:

(74) Sherlock pretended to either look for a burglar or a thief.
Readings: ✗ pretend>look for>or, ✓ pretend>or>look for, ✗ or>pretend>look for

Larson's generalization about the scope trapping ability of *either* is that whenever *either* is not adjacent to the disjunction it associates with, the disjunction is assigned scope at the level at which *either* occurs at surface form. To the extent this generalization is correct, and cannot be obviated by exceptional scope construals of disjunction, we can employ it to trap disjunction in our examples. More specifically, to the extent disjunction in the antecedent clause of (75) lacks an exceptional scope construal (say, the antecedent clause cannot be followed by "I don't remember which"), it

the licensing condition on *any* NPs (see Crnič and Buccola 2019 for a related discussion).

(ii) a. [neg [[any problem] [Gal studied [every analysis that solves ~~[any problem]~~]]]]
b. $\neg\exists x$: problem x $\wedge$ $\forall z$: analysis that solves problem x $\rightarrow$ Gal studied z

should not allow for an exceptional scope construal in (76) either, all else equal.

(75) If Gal either got into Cambridge or Oxford, he was lucky.

(76) Fewer than 1000 students have either got into Cambridge or Oxford.

If the scope of disjunction is indeed trapped under *fewer than 1000 students* in (76), the sentence should only have a low-scope disjunctive construal. In that case, it should be judged as false in our earlier scenario – in which more than 1000 students got into Cambridge or Oxford, though each univeristy admitted fewer than 1000 students individually. This prediction is borne out. More specifically, to the extent speakers fail to access an exceptional scope reading of sentence (75) – which indicates that the trapping with *either* is effective, as seems to generally be the case – they take sentence (76) to only convey that jointly Cambridge and Oxford accepted fewer than 1000 students. This contrasts with sentence (55) above, which allows for conjunctive strenghtening, as well as with a variant of sentence (76) in which *either* is adjacent to disjunction.[16,17]

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[16] A similar state of affairs holds also for sentences with *most*, as exemplified in (i): the sentence is judged as false in our scenario in the main text, in which 3 boys play for team A and B, 2 boys play just for team A, and 2 boys play just for team B (that is, only 3 of the 7 boys play for both teams).

(i) Most kids who either play for team A or team B play for both teams.

The sentence can nonetheless give rise to a conjunctive inference (see Bar-Lev and Fox 2020, Sect. 8.3, for a derivation), which means that the difference with respect to example (25) in the main text is merely in that (i) entails a low-scope disjunctive interpretation. Thanks to a reviewer for discussion.

[17] A related prediction arises for the variants of the examples in the main text in which *either* would attach at the matrix level. We discuss the prediction in relation to an occurrence of disjunction in a downward-monotone environment, with a major caveat that the sentence is somewhat marked for independent reasons (cf. Larson 1985 on *either* and negation, Schwarz 1999 on missing quantificational subjects in non-adjacent *either* disjunctions). Consider the sentence in (i) – to the extent the sentence is acceptable, it does not entail a conjunctive inference.

(i) ?Either fewer than 1000 students got into Cambridge or into Oxford.

This is in line with our proposal. Namely, Schwarz 1999 argues that sentences in which *either* is separated from its associate disjunction involve ellipsis (see also Wu 2022). Accordingly, since (i) involves such a separation, the sentence should have a structure like (iia). Importantly, the structure does not involve movement of disjunction, and should have a parallel conjunctive alternative, provided in (iib). This alternative should inhibit conjunctive strengthening, which would explain the absence of conjunctive strengthening. Thanks to a reviewer for discussion.

4.2 Non-exceptional scope taking indefinites

There are indefinites that do not allow for exceptional scope construal. This holds, in particular, for modified numeral indefinites (see, e.g., Reinhart 1997, Winter 2002, Ch. 3, for discussion, and Liu 1990, Beghelli 1995, Corblin 1997 for related earlier observations). For example, the sentence in (77a) cannot convey the meaning that there is a plurality consisting of more than one student such that everyone who knows all of the students in that plurality is lucky (see Ruys 1992, Reinhart 1997, Winter 1997 on the low-scope of distributivity with plural exceptional scope indefinites). This means that the sentence in (77a) lacks the LF in (77b).

(77) a. Everyone who knows more than one student is lucky.

b. #[more than 1 student] [everyone who knows ~~[more than 1 student]~~ is lucky]

Accordingly, we predict that *more than one* NPs cannot give rise to conjunctive inferences without contributing a low-scope existential meaning. For example, consider a scenario in which we have four 5-membered teams with three members in common and in which each of the two remaining kids on a team is on exactly two teams (thus, 4 of the 7 kids are on exactly 2 teams, 3 of the 7 are on all 4 teams). One configuration satisfying these conditions is in Figure 1.

kid 1: teams	A, B	
kid 2: teams	A, B	
kid 3: teams	A, B,	C, D
kid 4: teams	A, B,	C, D
kid 5: teams	A, B,	C, D
kid 6: teams		C, D
kid 7: teams		C, D

Figure 1: One possible make-up of the different teams.

The sentence in (78) cannot be judged true in this scenario. This means that it is impossible for the sentence to give rise to the conjunctive inference without the indefinite contributing a low-scope existential meaning to the interpretation of the sentence, as indicated in (79).

(ii) a. [either [fewer than 1000 students got into Cambridge] [or [~~fewer than 1000/pro got~~ into Oxford]]]

b. [fewer than 1000 students got into Cambridge] [and [~~fewer than 1000 students/pro got~~ into Oxford]]

(78) Most kids who are on more than one team are on all teams.

(79) **Observed inferences of sentence (78):**

a. $\Rightarrow$ $[\![\text{most}]\!](\{x \mid \text{kid x is on more than 1 team}\})(\{x \mid \text{kid x is on all teams}\})$

b. $\Rightarrow \forall X$: $[\![\text{two teams}]\!](X) \rightarrow [\![\text{most}]\!](\{x \mid \text{kid x is on X}\})(\{x \mid \text{kid x is on all teams}\})$

4.3 Scope, binding, and the analysis of indefinites

One standard method for trapping the scope of an indefinite involves binding a pronoun that is dominated by the indefinite (see, e.g., Schwarz 2001, Brasoveanu and Farkas 2011, Demirok 2019, Charlow 2020). In particular, an indefinite that contains a variable bound by another quantificational expression cannot take scope over that expression. In our cases, then, if an indefinite contains a variable bound by a quantifier within an island, its scope will likewise be confined to that island – precluding scoping out and subsequent conjunctive strengthening. This is schematized in (80):

(80) #[an NP ... pro_x ...] [... [$_{island}$ QP_x ... ~~[an NP ... pro_x ...]~~] ...]

All else equal, we expect that in such configurations conjunctive inferences are necessarily accompanied by a low-scope existential contribution of indefinites.

Challenge. A reviewer invites us to consider the sentence in (81) on the bound reading of the embedded pronoun. Together with the editor, they observe that the sentence can be judged as true in a scenario that parallels our earlier ones: there are 7 kids in total, 3 kids admire both of their parents, 2 kids admire only their father, and the other 2 kids admire only their mother. In short, the sentence seems to be able to exhibit our target entailment pattern, as stated in (82) – that is, conjunctive strengthening with no existential contribution of the indefinite in its surface position.

(81) Most kids who_i admire one of their_i parents admire both.

(82) **Possible reading of sentence (81):**

a. $\not\Rightarrow$ $[\![\text{most}]\!](\{x \mid \text{kid x admires x's mom or dad}\})(\{x \mid \text{kid x admires x's mom and dad}\})$

b. $\Rightarrow$ $[\![\text{most}]\!](\{x \mid \text{kid x admires x's mom}\})(\{x \mid \text{kid x admires x's mom and dad}\}) \wedge$
$[\![\text{most}]\!](\{x \mid \text{kid x admires x's dad}\})(\{x \mid \text{kid x admires x's mom and dad}\})$

While the availability of this reading may *prima facie* appear to be at odds with the predictions of our proposal, this appearance stems primarily from the simplified treatment of indefinites.

Dependent indefinites. Indefinites permit a type of reading we have not discussed so far – one on which the contribution of the indefinite varies with a c-commanding quantifier (see, e.g., Hintikka 1986, Kratzer 1998, Chierchia 2001, Winter 2002, 2004, Schlenker 2006). For example, Schlenker 2006 discusses the sentence in (83), which can describe a scenario in which there is a distribution of study areas among students such that, if every student improves in their designated area, no one will flunk the exam. Crucially, the sentence is false on its surface scope reading: if each student improves in an area in which they already excel, the antedent is true but the consequent may be false. It is likewise false on both wide-scope construals of the indefinite, as there is no single area such that if each student improves in it, nobody will flunk the exam (see Winter 2002, Schlenker 2006, Endriss 2009 for discussion and motivation). To capture the target reading here, a different construal of the sentence is required.

(83) [Context: Every student in my syntax class has one weak point – John doesn't understand Case Theory, Mary has problems with Binding Theory, etc. Before the final, I say:]
If each student makes progress in one area, nobody will flunk the exam.

It is controversial what the precise distribution of such readings is and what their proper analysis is (see, e.g., Ebert 2020 for a recent review). One influential analysis takes such indefinites to denote Skolemized choice functions (see Sect. 2.3 above on choice functions and notational conventions). Skolemized choice functions are functions from individuals to choice functions: for each individual, a Skolemized choice function picks out for it an element from a set of individuals (e.g., Winter 2002, 2004, Schlenker 2006). Accordingly, sentence (83) may be analyzed as in (84), yielding the target reading of the sentence, on which specific areas vary with students.

(84) a. [$\exists_f$ [if each student$_x$ makes progress in one$_{f(x)}$ area, nobody flunks the exam]]
b. ∃f∈SCH: if each student$_x$ makes progress in f(x)(area), nobody flunks the exam

Turning back to the sentence in (81), an LF that yields such a dependent interpretation is provided in (85a), where the parameter of the Skolem choice function is bound by the quantifier. The corresponding interpretation is provided in (85b), where the choice of a parent depends on the kid. Crucially, this interpretation does not entail the wide-scope reading of the indefinite, despite the fact that existential closure applies at the matrix level (see, e.g., Schwarz 2001, 2011 for discussion).

(85) a. $[\exists_f$ [most$_x$ kids who admire one$_{f(x)}$ of their$_x$ parents admire both their$_x$ parents$]]$

b. $\exists$f$\in$SCH: $[\![$most$]\!]$({x | kid x admires f(x)({x's mom, x's dad})

({x | kid x admires x's mom and dad})

If we assume that the Skolemized choice functions in the domain of existential closure must be 'natural' (cf. Sharvit 1999, Winter 2004, Schlenker 2006; but see, e.g., Endriss 2009 for qualifications), one can reformulate the meaning in (85) as in (86). In this formulation, the pertinent choice functions map each individual to their mother or to their father.

(86) (85) $\Leftrightarrow$ $[\![$most$]\!]$({x | kid x admires x's dad})({x | kid x admires x's mom and dad}) $\vee$

$[\![$most$]\!]$({x | kid x admires x's mom})({x | kid x admires x's mom and dad})

The set of alternatives to the structure in (85a) is not closed under conjunction, and the structure may be strengthened to convey a conjunctive inference according to condition (C): namely, the conjunction of the two proper subdomain alternatives is not equivalent to the low-scope existential alternative, as witnessed by the scenario provided above. Conjunctive strengthening is achieved by recursive exhaustification, as indicated in (87), where the crucial role is again played by the two subomain alternatives (which correspond to the exhaustifications of the two disjuncts in (86)): if one of them is true, the other one is as well, and since one must be true, both of them are. Crucially for our purposes, this construal does not entail a low-scope meaning of the indefinite.

(87) $[\![$[exh$_{C'}$[exh$_C$[$\exists_f$[most$_x$ kids who admire one$_{f(x)}$ of their$_x$ par. admire both their$_x$ par.]]$]\!]$ =

$[\![$most$]\!]$({x | kid x admires x's dad})({x | x admires x's mom and dad}) $\wedge$

$[\![$most$]\!]$({x | kid x admires x's mom})({x | x admires x's mom and dad})

We have thus seen that trapping by binding does not necessarily preclude the generation of conjunctive inferences without the indefinite contributing its meaning in its surface position. We outlined an approach to these data that treats the relevant indefinites as functional indefinites, and modeled this using Skolemized choice functions. If an analogous analysis is unavailable on the simple movement approach – something we cannot explore here – then the data discussed here may provide grounds for adopting a more sophisticated, choice function treatment of indefinites.

Fortunately, such a switch would leave the much of our preceding discussion intact (see Sect. 2.3).[18]

5 Wrapping up with conditionals

Our starting-point was a prediction of the grammatical theory of exhaustification and alternatives: when a sentence that dominates disjunction or an indefinite lacks a parallel conjunctive or universal quantifier alternative, its set of alternatives might fail to be closed under conjunction, and the sentence might accordingly give rise to a conjunctive inference. We explored this prediction by examining cases in which disjunction and indefinites take scope unavailable to their conjunctive and universal quantifier alternatives – specifically, scope out of islands and over certain downward-monotone operators. The prediction was borne out.

The paper thus contributes in two ways: (i) it broadens the range of constructions in which disjunction and indefinites can undergo conjunctive strengthening, and (ii) it provides further support for the explanatory potency of the grammatical theory of exhaustification and alternatives. Nevertheless, many questions and directions for further research remain. These include predictions concerning conjunctive strengthening in environments beyond those considered above. As a partial step in this direction, we discuss in Sect. 5.1 some predictions regarding the behavior of exceptional scope disjunction in the antecedents of conditionals (cf. Santorio 2018, 2020). We then conclude in Sect. 5.2 with a discussion of issues concerning the robustness and cancellability of conjunctive inferences.

5.1 Conjunctive strengthening in conditionals

The interpretation of conditional sentences depends on the modal expression in the matrix clause of the conditional (e.g., Kratzer 1981, 1986). Although the antecedents of conditionals typically constitute non-monotone environments (cf., e.g., Stalnaker 1968, Lewis 1973, Kratzer 2012; see von Fintel 2001 for a contrasting perspective), conditionals differ with respect to what inferences they give rise to if disjunction or indefinites appear in their antecedents. We zoom-in here on the behavior of disjunction in three types of conditionals, following Santorio 2018, 2020: *probably* conditionals,

[18] The reviewer's example in (81) is easily construed as having a dependent reading of the indefinite, and the functions in the domain of the existential closure are salient natural functions. It may well be that such construals always require salience of the pertinent natural functions in the context. Absent such salience, or natural functions altogether, we would then expect the conjunctive strengthening readings of the kind discussed in the main text not to be readily available.

deliberative conditionals, and bare conditionals. We predict conjunctive inferences for exceptional scope disjunction in *probably* and deliberative conditionals, which have indeed been observed by Santorio 2018, 2020. In contrast, we predict an exclusive inference for exceptional scope disjunction in bare conditionals, which has indeed been observed by Charlow 2019.

This distribution of inferences is governed by whether the conjunctive inferences in the sentences under discussion entail the respective low-scope disjunctive alternatives (which are excludable in all three kinds of conditionals), rather than by the non-closure under conjunction of the sets of alternatives – a simple and coarse criterion we employed so far (which does not distinguish the three kinds of conditionals). Thus, the otherwise puzzling data involving disjunction and indefinites in conditionals follows naturally on the grammatical approach to exhaustification – no revisions of the semantics of disjunction or conditionals are required to account for these facts.

(i) *Probably* conditionals

Santorio 2018 observes that a *probably* conditional sentence in (88) can convey the conjunctive inference without disjunction contributing a disjunctive meaning in its surface scope position, as stated in (89). More to the point, consider a situation in which Sarah buys 40 lottery tickets, namely, tickets from 31-70. When addressing the question whether Sarah won the lottery or lost it, we can judge sentence (88) to be true, even though its surface disjunctive interpretation would be false: namely, Sarah's 40 tickets constitutes the majority of tickets between 1-70, and between 31-100, but not between 1-100 (which is equivalent to between 1-70 or between 31-100; we assume that the total number of tickets is greater than 100).

(88) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

(89) **Possible reading of sentence (88):**

a. $\not\Rightarrow$ If the winning ticket is between 1 and 100, probably Sarah won

b. $\Rightarrow$ If the winning ticket is between 1 and 70, probably Sarah won $\wedge$
If the winning ticket is between 31 and 100, probably Sarah won

Before proceeding to the derivation, we should elaborate on the missing low-scope disjunctive meaning in (88), and on some complexities involving the interpretation of *probably* conditionals.

Missing low-scope disjunctive meaning and *probably*. A reviewer notes that weak construals of *probably* may be available in (88), that on such construals the low-scope disjunctive meaning of the sentence may be evaluated as true in the scenario, and hence that our claim that disjunction takes exceptional scope may not be warranted (see Lassiter 2011 for a review of the context-dependency of expressions of uncertainty, including data that shows that *probably* need not always convey 'more than 50% likely'). They suggest several ways of controlling for the strength of the conditional in order to solidify the judgements about the target reading. We have already tried to implement one of their suggestions by fixing the salient question under discussion in our scenarios. More to the point, Lassiter 2011 notes that the strong reading of *probably* is facilitated in examples like ours if the sentence addresses the question of whether Sarah won or lost the lottery (and not, say, who won the lottery). Hence, we insisted on mentioning this question in our setup. Furthermore, we can enrich the above scenario by assuming that another salient individual, Zara, purchased the remaining 60 of the first 100 tickets. B's response can still be evaluated as true, while C's response cannot be, as indicated in (90).[19]

(90) [Scenario as above. Zara bought the remaining 60 tickets, that is, tickets 1-30 and 71-100.]

A: Did Sarah win the lottery or did she not win the lottery?

B: If the winning ticket is between 1 and 70 or between 31 and 100,
probably Sarah won the lottery.

C: %If the winning ticket is between 1 and 100, probably Sarah won the lottery.

Finally, the reviewer suggests another control: employing a less context-sensitive expression than *probably*. Two sentences containing such expressions are provided in (91), both of which can be judged true in our scenario. In contrast, this is not the case for the paraphrases corresponding to low-scope disjunctive construals of the sentences in (92).

(91) The following sentences <u>can</u> be judged True in the above scenario:

a. If the winning ticket is between 1 and 70 or between 31 and 100,

[19] While we are trying to show that the pertinent sentences with *probably* conditionals are indeed false on their low-scope disjunctive construals by fixing its value to 50%, it should be noted that an argument analogous to Santorio's can be constructed for whatever value other than 50% one may employ in the evaluation of *probably* conditionals, simply by appropriately revising the numbers in our scenario. For illustration, if the pertinent value in the evaluation of *probably* was merely 30%, the tickets bought by Sarah could range between 45 and 70, etc.

more likely than not Sarah won.

b. If the winning ticket is between 1 and 70 or between 31 and 100,
it is more than 50% likely that Sarah won.

(92) The following sentences cannot be judged True in the above scenario:

a. If the winning ticket is between 1 and 100,
more likely than not Sarah won.

b. If the winning ticket is between 1 and 100,
it is more than 50% likely that Sarah won.

In short, the sentences under discussion give rise to conjunctive inferences, while not entailing a low-scope disjunctive reading of the sentence. This parallels our observations about the behavior of disjunction in the preceding sections. (For concreteness, we treat *probably* conditionals as expressing the proposition that the probability of the consequent holding conditional on the antecedent holding is greater than 50%, a value which may well vary by context, see, e.g., Yalcin 2010, Lassiter 2011 for a more sophisticated analysis, as well as fn. 19.)

Scope and strengthening. The sentence in (88) can be parsed with disjunction taking exceptional scope and the sentence being recursively exhaustified, as in (93).

(93) a. If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

b. [exh$_{C'}$ [exh$_C$ [$_S$ [between 1-70 or between 31-100]
[probably [if W is ~~[between 1-70 or between 31-100]~~] [Sarah won]]]]]

Given our assumptions, the formal alternatives to the base sentence, S in (93b), consist of two disjunct alternatives, a low-scope disjunctive alternative, and a low-scope conjunctive alternative:

(94) ALT(S) = { [probably [if W is [between 1-70 or between 31-100]] [Sarah won]],
[probably [if W is between 1-70] [Sarah won]],
[probably [if W is between 31-100] [Sarah won]],
[probably [if W is [between 1-70 and between 31-100]] [Sarah won]] }

In line with our preceding discussion, the sentence lacks an exceptional scope conjunctive alternative, of the form provided in (95), as such alternatives are not generated by grammar. This means that the set of alternatives to the sentence is not closed under conjunction, as stated in (96): for example, the conjunction of the disjunct alternatives is not equivalent to the low-scope disjunctive alternative (as witnessed by our initial scenario in which the former is true but the latter is false), or any other alternative for that matter. Accordingly, conjunctive strengthening may be possible.

(95) [between 1-70 and between 31-100]

[probably [if W is ~~[between 1-70 and between 31-100]~~] [Sarah won]]

$\notin$ ALT(S)

(96) **Condition (C) admits potential conjunctive inference for (88):**

ALT(S) is not closed under conjunction.

The excludable alternatives in (94) are the low-scope disjunctive alternative and the low-scope conjunctive alternative; neither disjunct alternative is excludable. If we assume that no low-scope alternative is relevant, we obtain the meaning in (97) for the embedded exhaustification:[20]

(97) $[\![\text{exh}_C \text{ S}]\!]$ = Pr(Sarah won | W is between 1-70) $> 0.5 \vee$

Pr(Sarah won | W is between 31-100) > 0.5

All the alternatives to the sister of the higher *exh* are excludable. If only the disjunct alternatives are relevant, matrix exhaustification yields the conjunctive inference, as provided in (98).[21]

[20]Unlike in the earlier examples, the assumption about pruning is not merely cosmetic here. In particular, if the alternatives are not pruned, conjunctive strengthening is inhibited – it cannot be that Sarah has more than half of the tickets between 1-70 and between 31-100, but not more than half of the tickets between 1-100 (low-scope disjunction) or between 31-70 (low-scope conjunction). The pruning is admitted by the extant conditions on pruning (e.g., it weakens the output of low exhaustification, cf. Crnič et al. 2015, Bar-Lev 2018).

[21]A different implementation of *exh*, say, that of Bar-Lev and Fox 2020, succeeds in a similar fashion. In particular, as we noted in the main text, the low-scope disjunctive and conjunctive alternatives to S in (93b) are excludable. This means that the disjunct alternatives are not includable according to Bar-Lev and Fox's 2020 definition: it cannot be that both disjunct alternatives are true, while both excludable alternatives are false (as discussed in fn. 20). But a recursive application of *exh* (with the pruning discussed in fn. 20) can deliver the target reading, by the exclusion of the exhaustified disjunct alternatives. This means that exhaustification in the main text and its variant with inclusion proceed fully in parallel in the example at hand, with only exclusion playing an active role in the derivation. In all the earlier examples in the main text, a single application of Bar-Lev and Fox's *exh* derives conjunctive inferences. See, e.g., Bar-Lev and Fox 2020, Alxatib 2023, Degano et al. 2023 for a further comparison of the different formulations of *exh*.

(98) $[\![\text{exh}_{C'}\ [\text{exh}_C\ \text{S}]]]\!] = \Pr(\text{Sarah won} \mid \text{W is between 1-70}) > 0.5\ \wedge$

$$\Pr(\text{Sarah won} \mid \text{W is between 31-100}) > 0.5$$

(ii) Deliberative conditionals

Santorio 2020 discusses sentence (99), an instance of a deliberative conditional, in relation to the following scenario (cf., e.g., Thomason 1981, Schroeder 2011, Cariani et al. 2013 on deliberative modal flavor): A six-sided die has been thrown and you are offered a bet on even or on odd. The bet pays \$50 if you win, and costs you \$75 if you lose. Gal tells you that the die landed on 2 or 4. Tal tells you that it landed on 3 or 5. You do not know whether either report is accurate. Given this scenario, and as further elaborated on below, Santorio reports that sentence (99) can be judged both true and offering sound advice.

(99) If Gal's report or Tal's report is accurate, you ought to place a bet.

Conjunctive and missing low-scope disjunctive meaning. On the reading under discussion, sentence (99) does not have a surface disjunctive interpretation, as stated in (100a). Namely, you should not place a bet if all that is known is that one of Gal and Tal is correct – that is, that the die landed on a number between 2 and 5 – since, all else equal, you are likely to lose more than you could gain (see Santorio 2020 for further discussion). On top of that, the sentence gives rise to the conjunctive inference in (100b).

(100) **Possible reading of sentence (99):**

a. $\not\Rightarrow$ If the die landed on a number between 2 and 5, you ought to place a bet

b. $\Rightarrow$ If the die landed on 2 or 4, you ought to place a bet $\wedge$

If the die landed on 3 or 5, you ought to place a bet

Scope and strengthening. The sentence in (99) can be parsed so that disjunction takes exceptional scope and the sentence is recursively exhaustified, as in (101).

(101) [exh$_{C'}$ [exh$_C$ [GR or TR] [ought [~~[GR or TR]~~ is accurate] [you place a bet]]]]

The conjunction of the disjunct alternatives is not equivalent to the low-scope disjunctive alternative, in particular, the conjunction of the disjunct alternatives does not entail the low-scope disjunctive alternative. This is supported by us accepting the sentence in (99) in the scenario described above, and rejecting its low-scope disjunctive variant, as summarized in (100a). Since the sentence lacks an exceptional scope conjunctive alternative, this means that conjunctive strengthening of the sentence may be possible according to condition (C).

(102) **Condition (C) admits potential conjunctive inference for (99):**
ALT([GR or TR] [ought [~~[GR or TR]~~ is accurate] [you have place a bet]]])
is not closed under conjunction.

Unlike in the case of *probably* conditionals, however, it is controversial why precisely the low-scope disjunctive alternative does not follow from the conjunction of the disjunct alternatives (see, e.g., Kolodny and MacFarlane 2010 for related discussion). The theoretical details are not crucial for our purposes, though we sketch for concreteness a derivation that builds on the restrictor approach to conditionals. On this approach, what distinguishes deliberative *ought* conditionals from 'regular' *ought* conditionals is that the former are sensitive to an additional parameter, a contextually salient decision problem (that is, a set of actions available to the deliberating agent), which affects the ordering of the worlds (Cariani et al. 2013). This additional parameter effectively makes the worlds in the modal base in which a specific action in the decision problem is chosen tied with respect to their goodness, and this goodness is determined on the basis of worlds in the modal base in which the specific action yields the worst outcome. In our example, the decision problem consists of three actions: bet on even, bet on odd, and don't bet. With respect to the conjunctive inference, we get that the optimal worlds in which Gal's report (2,4) is accurate are those in which you bet on even, and the optimal worlds in which Tal's report (3,5) is accurate are those in which you bet on odd. In all of these worlds, you indeed place a bet. In contrast, with respect to the low-scope disjunctive alternative, the optimal worlds in which Gal's or Tal's report (2-5) is accurate are those in which you do not place a bet (= you do not lose any money) since those are better than the worlds in which Tal's report (3,5) is accurate and you bet on even (= you lose \$75), and those in which Gal's report (2,4) is accurate and you bet on odd (= you lose \$75) – recall that this comparison is dictated by having to effectively look only at the worlds in the modal base in which a specific action compatible with the antecedent yields the worst outcome (see Cariani et al. 2013 for details and motivation). This means that the conjunction of the disjunct alternatives does not entail alternatives, as stated in (103) (where $\text{MIN}_{\leq}^{Dec}$ is meant to pick out the optimal worlds given the decision problem *Dec*). Accordingly, condition (C) admits strengthening on the proposed analysis of deliberative conditionals.

(103) $\textsc{min}^{Dec}_{\leq}(\{w \mid \text{Gal's report is accurate in } w\}) \subseteq \{w \mid \text{you place a bet in } w\} \wedge$
$\textsc{min}^{Dec}_{\leq}(\{w \mid \text{Tal's report is accurate in } w\}) \subseteq \{w \mid \text{you place a bet in } w\}$

$\not\Rightarrow \textsc{min}^{Dec}_{\leq}(\{w \mid \text{G or T's report is accurate in } w\}) \not\subseteq \{w \mid \text{you place a bet in } w\}$

Recursive exhaustification delivers this strengthening. At the first layer of exhaustification, we obtain the meaning in (104) if the excludable low-scope disjunctive alternative is not relevant (unlike in the case of *probably* conditionals, this assumption is merely cosmetic here, see fn. 20).

(104) $[\![$[exh$_C$ [GR or TR] [ought [~~GR or TR~~ is accurate] [you place a bet]]$]\!]$ =

$\textsc{min}^{Dec}_{\leq}(\{w \mid \text{Gal's report is accurate in } w\}) \subseteq \{w \mid \text{you place a bet in } w\} \vee$
$\textsc{min}^{Dec}_{\leq}(\{w \mid \text{Tal's report is accurate in } w\}) \subseteq \{w \mid \text{you place a bet in } w\}$

At the second layer of exhaustification, the exhaustified disjunct alternatives are excludable. This is witnessed by their joint exclusion being consistent, as indicated in (105) (with the exclusion of the exhaustified disjunct alternatives, we obtain the inference that if one of the disjunct alternatives is true, the other one is as well). This meaning corresponds to the target conjunctive inference.

(105) $[\![$[exh$_{C'}$ [exh$_C$ [GR or TR] [ought [~~GR or TR~~ report is accurate] [you place a bet]]$]\!]$ =

$\textsc{min}^{Dec}_{\leq}(\{w \mid \text{Gal's report is accurate in } w\}) \subseteq \{w \mid \text{you place a bet in } w\} \wedge$
$\textsc{min}^{Dec}_{\leq}(\{w \mid \text{Tal's report is accurate in } w\}) \subseteq \{w \mid \text{you place a bet in } w\})$

In conclusion, the observed conjunctive inferences of *probably* and deliberative conditionals were derived on the grammatical theory of exhaustification. Accordingly, the complex behavior exhibited by disjunction in conditionals does not require a revision of the analysis of conditionals or disjunction (*pace* Santorio 2018, 2020).

(iii) Bare conditionals

We conclude the discussion of conditionals by looking at the bare conditional sentences in (106). Charlow 2019 notes that if we assign exceptional scope to the disjunction in it, the sentence gives rise to the inference that exactly one of cake or salad is such that if Gal orders it, her parents are relieved. That is, the sentence gives rise to the negation of the conjunctive inference – which is the opposite of what we observed for *probably* and deliberative conditionals. (On the default, low-scope construal of disjunction, the sentence gives rise to a conjunctive inference, which is called 'Simplification of Disjunctive Antecedents', see Bar-Lev and Fox 2020, among many others.)

(106) If Gal ordered cake or salad, her parents are relieved.

The sentence can be assigned the structure in (107), in which disjunction takes exceptional scope and the sentence is exhaustified. The sister of *exh* has the set of alternatives provided in (108).

(107) [exh$_C$ [$_S$ [cake or salad] [if Gal ordered ~~[cake or salad]~~ her parents are relieved]]]

(108) ALT(S) = { [cake or salad] [if Gal ordered ~~[cake or salad]~~ her parents are relieved],
[if Gal ordered [cake or salad] her parents are relieved],
[if Gal ordered cake her parents are relieved],
[if Gal ordered salad her parents are relieved],
[if Gal ordered [cake and salad] her parents are relieved] }

In contrast to the universal quantification sentence whose inferences we discussed in Sect. 2.1, the set of alternatives in (108) is not closed under conjunction: in particular, the conjunction of the disjunct alternatives is not equivalent to any alternative in the set. Let us see why. We treat conditionals as conveying that the worlds closest to the actual one in which the antecedent holds the consequent holds as well (cf., e.g., Stalnaker 1968, Lewis 1973, Kratzer 1981). While the conjunction of the disjunct alternatives clearly entails the low-scope disjunctive alternative, the two are not equivalent, as stated in (109). Namely, imagine a setup in which only salad is such that if Gal ordered it, her parents are relieved, and in which Gal ordering salad holds in all the closest worlds in which Gal orders something. In this setup the conjunction of the disjunct alternatives is false, while the low-scope disjunctive alternative is true – hence, the latter does not entail the former.

(109)
$$\begin{aligned}&\textsc{min}_{\leq}(\{w \mid \text{Gal ordered}_w \text{ cake}\}) \subseteq \big(\{w \mid \text{Gal's parents are relieved}_w\} \wedge \\ &\quad \textsc{min}_{\leq}(\{w \mid \text{Gal ordered}_w \text{ salad}\}) \subseteq \{w \mid \text{Gal's parents are relieved}_w\}\big) \\ &\quad {\Rightarrow}/{\not\Leftarrow}\ \textsc{min}_{\leq}(\{w \mid \text{Gal ordered}_w \text{ cake or salad}\}) \subseteq \{w \mid \text{Gal's parents are relieved}_w\}\end{aligned}$$

Given that the conjunction of the disjunct alternatives is not equivalent to any other alternative either, we conclude that indeed (108) is not closed under conjunction.

(110) **Condition (C) admits potential conjunctive inference for (106):**

ALT([cake or salad] [if Gal ordered ~~[cake or salad]~~ her parents are relieved])

is not closed under conjunction.

But a potential conjunctive inference being in principle admitted does not mean that it can in fact be generated, that is, conditon (C) characterizes merely a necessary condition on the generation of conjunctive and universal inferences. Whether conjunctive strengthening can obtain hinges on the nature of exhaustification, in particular, on whether the conjunctive inference entails any excludable alternatives to the sentence. The sentence in (106) has two excludable alternatives: the low-scope conjunctive and the low-scope disjunctive alternative, as stated in (111).

(111) IE([cake or salad] [if Gal ordered ~~[cake or salad]~~ her parents are relieved]) =

$$\Big\{\textsc{min}_{\leq}(\{\text{w} \mid \text{Gal ordered}_w \text{ cake or salad}\}) \subseteq \{\text{w} \mid \text{Gal's parents are relieved}_w\},$$
$$\textsc{min}_{\leq}(\{\text{w} \mid \text{Gal ordered}_w \text{ cake and salad}\}) \subseteq \{\text{w} \mid \text{Gal's parents are relieved}_w\}\Big\}$$

The excludability of the low-scope disjunctive alternative is inhibitory with respect to conjunctive strengthening: the target conjunctive inference entails it, as stated in (109), so its exclusion entails the negation of the conjunctive inference, which consequently cannot be generated. Instead, the negation of the excludable disjunctive alternative entails the negation of the conjunctive inference, which corresponds to the target inference of sentence (106), namely, that not both cake and salad are such that if Gal ordered it, her parents are relieved, as provided in (112) (specifically, the negation of the excludable disjunctive alternative entails that some of the closest worlds in which a disjunct holds are such that Gal's parents are not relieved; consequently, the corresponding disjunct alternative to the matrix sentence is also false, which in turn leads to the negation of the conjunctive inference).

(112) $[\![$[exh$_C$ [$_S$ [cake or salad] [if Gal ordered ~~[cake or salad]~~ her parents are relieved]]]]$]\!]$ $\Rightarrow$

$$\neg\Big(\textsc{min}_{\leq}(\{\text{w} \mid \text{Gal ordered}_w \text{ cake}\}) \subseteq \{\text{w} \mid \text{Gal's parents are relieved}_w\} \wedge$$
$$\textsc{min}_{\leq}(\{\text{w} \mid \text{Gal ordered}_w \text{ salad}\}) \subseteq \{\text{w} \mid \text{Gal's parents are relieved}_w\}\Big)$$

The preceding discussion illuminated three aspects of the theory. First: While condition (C) provides a good approximation of the distribution of conjunctive inferences, and is a helpful guide when it rules out conjunctive strengthening, it is merely a necessary condition – whether conjunctive strengthening is indeed predicted for specific cases requires the computation of the exhaustified meaning. Second: Conjunctive strengthening is tightly constrained by the alternatives to a sentence, an observation that we have already discussed in relation to universal quantification sentences in Sect. 2.1. Third: The inferences accompanying exceptional scope elements, their scalar implicatures, can be derived without assuming that the exceptional scope disjunction or indefinite have exceptional scope conjunction or universal quantifier as alternatives. This is reassuring given our characterization of alternatives in (5) and given that these latter alternatives cannot be generated in grammar (see, esp., Demirok 2019, Charlow 2020 for two recent principled explanations of this).

Summary of the typology

We identified two types of conditionals in relation to what kinds of inferences disjunction that surfaces in their antecedents gives rise to. This typology is conditioned by the logical properties of the antecedents (cf. Santorio 2018, 2020). It has been previously established that if disjunction is interpreted as taking low-scope in conditionals, conjunctive strengthening obtains across the board, yielding so-called 'Simplification of Disjunctive Antecedents' inferences (see Bar-Lev and Fox 2020 for discussion and derivation). In contrast, if disjunction is interpreted as taking exceptional scope outside of the antecedent, only the *probably* and deliberative conditionals can give rise to conjunctive inferences since bare conditionals induce alternatives that inhibit such strengthening.

(113) a. ***Probably* and deliberative conditionals with disjunctive antecedents:** exceptional scope construal of disjunction leads to conjunctive strengthening (and due to its exceptional scope there is no low-scope disjunctive contribution of disjunction)

b. **Bare conditionals with disjunctive antecedents:** exceptional scope construal of disjunction leads to an exclusive inference (and due to its exceptional scope there is no low-scope disjunctive contribution of disjunction)

5.2 Optionality vs. obligatoriness of conjunctive strengthening

In many cases discussed above, conjunctive strengthening does not appear to be obligatory. The availability of non-conjunctive readings can be demonstrated by appending continuations such as "I don't remember which," which are incompatible with conjunctive strengthening. For example, such a continuation is felicitous in (114), but not in its variant with *either* in (115).

(114) If the winning ticket is between 1-70 or between 31-100, probably Sarah won.

I don't remember which.

(115) #If the winning ticket is from either grouping, probably Sarah won. I don't remember which.

While the unacceptability of (115) is expected – conjunctive strengthening is required in the first sentence due to the licensing condition on *either* and is incompatible with the continuation (see fn. 14) – the acceptability of (114) is more puzzling. Although this might initially seem unproblematic, given that exhaustification is not always obligatory, the issue becomes pressing if we assume, as one arguably should, that conjunctive strenghtening applies whenever it is possible. This assumption is stated in its simplest form in (116) (see Chemla and Singh 2014, Bar-Lev and Fox 2020, Gotzner and Romoli 2022 for extensive discussion and refinements; see fn. 9 for related discussion).

(116) **An approximation of a generalization:**
If a sentence has a parse that can be conjunctively strengthened, conjunctive strenghtening must apply to that parse.

To satisfy this generalization, sentences that have both a conjunctive strengthening reading and a non-conjunctive one have been analyzed as having two substantially different parses. For example, consider the sequence in (117): the first sentence tends to give rise to a conjunctive inference when presented on its own, but the inference can be suspened by a continuation like "I don't know which."

(117) You are allowed to have cake or soup. I don't know which.

The disappearance of the conjunctive inference in (117) has been attributed to the first sentence being parsed with wide-scope disjunction (see, e.g., Fusco 2019), as provided in (118) (the sentence may then even be strengthened to convey an exclusive meaning, assuming that the wide-scope conjunctive alternative is available here).

(118) [cake or soup] [you are allowed to have ~~[cake or soup]~~]

Thus, the acceptability of (117) is compatible with the generalization in (116): conjunctive strengthening simply cannot apply to the parse in (118), so it does not. This strategy is not obviously available to us in (114), however. Namely, we are already assigning the widest scope to disjunction. In this specific case, however, the target non-conjunctive reading can be derived simply by not pruning the low-scope disjunctive alternative in exhaustification (which is crucial for deriving conjunctive strenghtening, see fn. 20). But what about cases in which such pruning is of no avail – for example, the first sentence in (119)?

(119) Most kids who are on team A or team B are on both teams. I don't know which.

Here is a possible direction. Let's assume that instead of the condition in (116), a slightly different condition governs strenghtening: each sentence must be recursively exhaustified, and the pruning of disjunct alternatives is dispreferred (see Bar-Lev and Fox 2020 for a more sophisticated condition).

(120) **A different approximation of a generalization:**
Every sentence must be c-commanded by recursively applied exhaustification operators.

Now, what structures can we assign to the sentence in (119), in which disjunction takes exceptional scope? At least two possibilities present themselves. One has been discussed at length in this paper. The other is illustrated in (121), which differs from the parses above merely in having a silent ASSERT operator – a prefix that corresponds to a universal epistemic modal, conveying that the speaker believes the sentence (see, e.g., Alonso-Ovalle and Menéndez-Benito 2010, Chierchia 2013, Meyer 2013, Cohen and Krifka 2014, Krifka 2014, Beck 2016, among many others).

(121) [ASSERT [[teamA or teamB] [most kids who are on ~~[teamA or teamB]~~ are on both teams]]]

Recursive exhaustification of this structure – assuming that ASSERT lacks an existential modal alternative – does not yield a conjunctive inference. Instead, it yields the inference that the speaker is not certain which of the two teams is such that most kids on it are on both teams, as shown in (122) (cf. Bar-Lev and Fox 2020, Sect. 5.5, for related discussion, and Degano et al. 2023 for qualifications).

(122) $\Box$(most kids on team A are on both teams $\vee$ (most kids on team B are on both teams) $\wedge$
$\neg\Box$(most kids on team A are on both teams) $\wedge$ $\neg\Box$(most kids on team B are on both teams)

Under this analysis, the continuation in (119) becomes compatible with the inferences of the first sentence. Whether this approach to the apparent cancellation of conjunctive strengthening in the examples at hand is ultimately tenable remains a question for future research.

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