Conjunctive strengthening more broadly

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Disjunctive sentences can sometimes be strengthened to convey conjunctive inferences. The same holds for sentences with indefinites. This remarkable behavior has been captured on the grammatical theory of exhaustification and alternatives, on which strengthening is modulated by what alternatives to sentences grammar generates. We examine a series of predictions of the theory stemming from one aspect of the grammar of disjunction and indefinites – their ability to undergo scope shift in certain cases in which their alternatives, conjunction and universal quantifiers, cannot. We show that since the structures with disjunction and indefinites in these cases, accordingly, lack parallel conjunctive and universal quantifier alternatives, this may enable exhaustification to generate conjunctive inferences for them. We show that this indeed occurs, expanding on some earlier observations of Santorio 2018, 2020 and Bar-Lev and Fox 2020.

1 Conjunctive strengthening

Disjunctive sentences can sometimes give rise to conjunctive inferences. This is exemplified in (1): the sentence in which disjunction occurs in the scope of an existential modal conveys the conjunctive inference that Jo may talk to Gal and that she may talk to Tal, the so-called 'free choice inference'.

(1) Jo may talk to Gal or Tal.

Conjunctive inference: $\Diamond$(Jo talks to Gal) $\wedge$ $\Diamond$(Jo talks to Tal)

Sentences with indefinites can also sometimes give rise to conjunctive inferences. This is exemplified in (2): the sentence conveys the conjunctive inference corresponding to the conjunction of sentences in which the indefinite of the initial sentence is replaced by a student name. (This meaning can be equivalently represented with universal quantification over students, and is more commonly called a 'universal inference'. We stick to the name 'conjunctive inference' for uniformity.)

(2) Jo may talk to a(ny) student.

Conjunctive inference: $\bigwedge_{x \in [\![\text{student}]\!]} \Diamond$(Jo talks to x)

$\big($*a.k.a. Universal inference:* $\forall$x: student x $\rightarrow \Diamond$(Jo talks to x)$\big)$

In contrast, simple disjunctive and indefinite sentences, like those in (3) and (4), not only fail to give rise to such inferences, they tend to give rise to the negation of conjunctive inferences, which more commonly go under the names of 'exclusive inference' and '*not all* inference'.

(3) Jo talked to Gal or Tal.

Negated conjunctive inference: $\neg$(Jo talked to Gal $\wedge$ Jo talked to Tal)

(4) Jo talked to a student.

Negated conjunctive inference: $\neg \bigwedge_{x \in [\![\text{student}]\!]}$ (Jo talked to x)

This complex behavior of disjunction and indefinites seems *prima facie* paradoxical.

Formal alternatives. Fox 2007 shows, however, that the behavior can be explained by recourse to exhaustification in grammar, an operation that quantifies over formal alternatives to sentences and strengthens their meaning. Since the outputs of strengthening are in the cases at hand conjunctive inferences, we call it 'conjunctive strengthening'. Furthermore, Fox shows that recourse to formal alternatives alone already suffices for describing the complex behavior of disjunction and indefinites. In order to state this description, however, we must first define the notion of a formal alternative. Formal alternatives to a sentence are syntactic objects that are at most as complex as the sentence, that is, they can be characterized as objects derived from the initial sentence by applying a series of substitutions or deletions to it (see Katzir 2007, Fox and Katzir 2011 for details, qualifications, and motivation). In addition, it is natural to assume that these objects are generated in grammar, that is, if an object is related to a sentence by means of appropriate deletions and substitutions, but cannot be generated in grammar, it does not count as an alternative to that sentence. In this paper, we hone-in on some consequences of this latter assumption, which we underline in the characterization of the alternatives in (5) (see fn. 12 on the generation of alternatives).

(5) **Formal alternatives:**

ALT(S) = {S′ | S′ is an object generated in grammar that can be derived from S by deletions and by substitutions of constituents with elements from the lexicon of the same category}

(Non-)closure under conjunction. The feature that robustly distinguishes between disjunctive and indefinite sentences that can give rise to conjunctive inferences from those that cannot is whether the sets of formal alternatives to a sentence are closed under conjunction (Fox 2007): if they are closed under conjunction, conjunctive strengthening is impossible; if they are not closed under conjunction, conjunctive strengthening may be possible (see also Chemla 2009, Franke 2011 for related discussion, and Singh et al. 2016 for an extensive review). (Although this condition approximates the necessary condition on the distribution of conjunctive inferences well-enough for the purposes of this paper, it can be further sharpened, as we discuss in Sect. 5.1.)

(C) **Condition of non-closure under conjunction (cf. Fox 2007):**

Conjunctive strengthening of a disjunctive sentence or a sentence with an indefinite, S, may obtain only if the set of formal alternatives to it, ALT(S), is not closed under conjunction.

Let's see how the condition applies to the sentences above. The existential modal sentence in

(1) induces the set of alternatives in (6), which consists of the initial sentence (alternative 1), of two disjunct alternatives (alternatives 2 and 3), and of the conjunctive alternative (alternative 4). (We follow the convention of leaving out alternatives that are equivalent to those provided as well as those that are not relevant for the discussion at hand, say, *Bo may talk to Jo*. We also do not quantifier raise quantificational expressions, incl. indefinites and coordinated NPs, just for interpretability reasons, but rather assume that type-shifting applies in the usual ways in those cases.)

(6) ALT(Jo may talk to Gal or Tal) =
{Jo may talk to Gal or Tal, Jo may talk to Gal,
Jo may talk to Tal, Jo may talk to Gal and Tal}

The set in (6) is <u>not closed</u> under conjunction since the conjunction of the disjunct alternatives is not equivalent to any other alternative in the set. For example, it is not equivalent to the conjunctive alternative: the conjunction of the disjunct alternatives may be true (Jo may talk to Gal and Jo may talk to Tal), while the conjunctive alternative is false (Jo may not both talk to Gal and Tal). Accordingly, condition (C) correctly allows for the sentence to give rise to a conjunctive inference.

(7) **Condition (C) admits potential conjunctive inference for (1):**
ALT(Jo may talk to Gal or Tal) is <u>not closed</u> under conjunction.
$\big($esp., $[\![$Jo may talk to Gal$]\!] \wedge [\![$Jo may talk to Tal$]\!] \not\Leftrightarrow [\![$Jo may talk to Gal and Tal$]\!]\big)$

The simple indefinite sentence like (4) induces alternatives analogous to (6), where we for simplicity assume that Gal and Tal are the only relevant students: in addition to the initial sentence and its universal quantifier alternative (alternatives 1 and 4), the sentence induces alternatives in which the domain of the indefinite or the universal quantifier in these two alternatives (that is, alternatives 1 and 4) is replaced with its subdomains (hence, the subdomain alternatives). Now, if the initial domain consists merely of Gal and Tal, as we are assuming, the subdomain alternatives correspond simply to the two disjunct alternatives from above.

(8) ALT(Jo talked to $\text{a}_{\{g,t\}}$ student) =
{Jo talked to $\text{a}_{\{g,t\}}$ student, Jo talked to Gal,
Jo talked to Tal, Jo talked to $\text{every}_{\{g,t\}}$ student}

In contrast to above, this set is <u>closed</u> under conjunction, as stated in (9): the conjunction of the

two subdomain alternatives corresponds to the universal quantifier alternative, the conjunction of the initial sentence and a subdomain alternative corresponds to the subdomain alternative, etc. According to condition (C), then, sentence *Jo talked to a student* cannot generate a conjunctive inference.[1]

(9) **Condition (C) precludes potential conjunctive inference for (4):**

ALT(Jo talked to $a_{\{g,t\}}$ student) is closed under conjunction.

$\big($esp., $[\![$Jo talked to Gal$]\!] \wedge [\![$Jo talked to Tal$]\!] \Leftrightarrow [\![$Jo talked to every$_{\{g,t\}}$ student$]\!]\big)$

Corroborated prediction. Strikingly, condition (C) can be satisfied even in simple disjunctive and indefinite sentences: namely, in languages in which disjunctive markers lack conjunctive marker alternatives, or indefinites lack universal quantifier alternatives, the sets of alternatives to simple disjunctive and indefinite sentences are not closed under conjunction (since conjunction of the disjunct and subdomain alternatives are not equivalent to any other alternatives in such cases):

(10) **Missing lexical alternatives:**

a. A simple disjunctive sentence in a language that lacks a conjunctive marker satisfies condition (C), and may hence be able to give rise to a conjunctive inference.

b. A simple indefinite sentence in a language that lacks a universal quantifier satisfies condition (C), and may hence be able to give rise to a conjunctive inference.

This prediction has sparked a lively research program in which numerous arguments have been brought forward in support of it across a host of different constructions, across a host of different languages, that is, languages in which disjunctive markers lack conjunctive marker alternatives, and in which indefinites or other existential quantifiers lack universal quantifier alternatives (see, e.g., Davidson 2013, Bowler 2014, Bar-Lev and Margulis 2014, Singh et al. 2016, Bassi and Bar-Lev 2016, Tieu et al. 2016, 2017, Bar-Lev 2018, Staniszewski 2021, Jeretič 2021; but see Haslinger and Schmitt 2019 for a critical discussion of some of these proposals).

[1] If the indefinite ranges over more than just two individuals, the closure of the set of alternatives to sentences like the (4) is conditioned by the sentences inducing also plural alternatives. For example, if we assume that the domain of the initial indefinite consists of four students, {g,t,u,v}, the conjunction of $[\![$Jo talked to $a_{\{g,t\}}$ student$]\!]$ and $[\![$Jo talked to every$_{\{u,v\}}$ student$]\!]$ is, for example, equivalent to $[\![$Jo talked to some$_{\{g+u+v,\ t+u+v\}}$ students$]\!]$, etc. A similar state of affairs obtains with disjunctions that have more than two disjuncts.

More general prediction. The prediction in (10) can plainly be generalized beyond simple sentences and missing lexical alternatives: if a sentence contains disjunction or an indefinite, and this sentence lacks a parallel conjunctive or universal quantifier alternative, for whatever reason, it may be able to give rise to a conjunctive inference due to the set of alternatives to the sentence potentially not being closed under conjunction. Now, this generalization, summarized in (11), is substantial to the extent there are sentences containing disjunction or indefinites that lack parallel conjunctive or universal quantifier alternatives for reasons other than the language having a depleted lexicon.

(11) **Missing alternatives *simpliciter*:**

a. A sentence that dominates disjunction, ϕ[S or S'], and lacks a parallel conjunctive alternative, *ϕ[S or S'/S and S'], may satisfy condition (C), and may hence be able to give rise to a conjunctive inference, $[\![\phi[\text{S or S'/S}]]\!] \wedge [\![\phi[\text{S or S'/S'}]]\!]$.

b. A sentence that dominates an indefinite, ϕ[an NP], and lacks a parallel universal quantifier alternative, *ϕ[an NP/every NP], may satisfy condition (C), and may hence be able to give rise to a conjunctive inference, $\bigwedge_{x \in [\![\text{NP}]\!]} [\![\phi[\text{an NP}]]\!]^{[an\ NP \rightarrow x]}$.

Here is one schema for constructing such sentences that does not hinge on a depleted lexicon: if some grammatical operation can target disjunction or an indefinite, but cannot target conjunction or a universal quantifier, an application of this operation allows one to generate sentences with disjunction or indefinites (namely, sentences in which the operation has applied to disjunction or indefinites) that do not have parallel conjunctive or universal quantifier alternatives (namely, the operation cannot apply to conjunction or universal quantifiers). This is stated in a slightly more general form in (12):

(12) **A schematic consequence of the constraint in (5):**

If expresssion α has an alternative β, $\beta \in \text{ALT}(\alpha)$, and operation OP can apply to α in sentence S to yield a well-formed structure, OP(α, S[α]), but it cannot do so to β in corresponding sentence S in which β replaces α, *OP(β, S[α/β]), then the former, well-formed structure lacks the latter structure as an alternative, *OP(β, S[α/β]) $\notin$ ALT(OP(α, S[α])).

Preview. The proposal that alternatives are indeed constrained in the way predicted by the characterization in (5) can be corroborated (or falsified) by what inferences pertinent sentences give rise

to.[2,3] In this paper, we discuss several instances of the schema in (12), all of which exploit asymmetries in the scope shifting abilities of disjunction and indefinites vs. conjunction and universal quantifiers. In Sect. 2, we discuss scoping out of islands, which is possible for disjunction and indefinites – this is their 'exceptional scope' taking ability – but not for conjunction and universal quantifiers (e.g., Fodor and Sag 1982, Rooth and Partee 1982). Specifically, we zoom in on one class

[2]The task of figuring out what alternatives a sentence may have, and what inferences it can accordingly give rise to, is often nontrivial. For example, one structure in which two scalemate items have a famously complementary distribution is the existential construction, as exemplified in (i). This should in principle make it a good candidate for testing (12).

(i) a. There are some students from my class in the garden.
b. *There are all students from my class in the garden.

Now, sentence (ia) can give rise to the scalar implicature that not all students from my class are in the garden. This appears to be derived on the basis of the ungrammatical (surface) alternative to the sentence, provided in (ib). Does this present an issue for the formulation in (5) and its consequence in (12)? The answer is 'not necessarily'. For example, the existential construction in (ia) may well have the complex structure in (iia), as argued by, e.g., Chomsky 1995, Lasnik 1995. It should thus have the well-formed alternative in (iib). And the negation of this alternative could then be the source of the observed scalar implicature. See also fn. 12 for a different way of getting the alternative in (iib).

(ii) a. [some students from my class [there are ~~[some students from my class~~ in the garden]]]
b. [all students from my class are in the garden]

[3]A reviewer points to some underappreciated observations by Schmitt 2013 as potential candidates for giving rise to conjunctive inferences due to missing conjunctive alternatives. Two examples are provided in (i), both of which give rise to a conjunctive inference (Schmitt 2013, p. 226):

(i) a. Ich esse (gern) Karotten, Bohnen oder (auch) Gurken. [German]
'I (like to) eat carrots, beans or (also) cucumbers.'
b. Geboten werden lange Artikel, Analysen und Abhandlungen über Geschichte, Wissenschaft und Politik. Sigmund Freud, Albert Einstein oder Bill Clinton haben für die Britannica geschrieben. [German]
'The Encyclopedia Britannica offers in-depth articles, analysis and treatments on history, science and politics. Sigmund Freud, Albert Einstein or Bill Clinton contributed to the Britannica.'

An analysis of the sentences in (i) that assigns them parses in which a (covert) existential operator would c-command disjunction, so that they would resemble sentence (1) in the main text, seems ill-motivated. Hence, finding a source of the conjunctive inferences of (i) in them missing conjunctive alternatives may well be promising. An eventual account of these data might also be extendable to the infamous examples of 'subtrigged' *any* NPs that are acceptable and give rise to conjunctive inferences without, apparently, being c-commanded by appropriate operators, exemplified in (ii).

of examples of exceptional scope shift out of non-monotone environments, for which conjunctive inferences are correctly predicted. In Sect. 3, we discuss examples in which disjunction and indefinites take scope out of certain downward-monotone environments, a scope shift that is unavailable to conjunction and universal quantifiers (e.g., Mayr and Spector 2012, Fleisher 2015). Specifically, we zoom in on two classes of examples of scope shift out of downward-monotone environments. Again, we show that conjunctive inferences are correctly predicted. In Sect. 4, we turn to some of the dauntingly many predictions of the proposal. We close the paper in Sect. 5 with the discussion of conjunctive inferences in conditionals, building on observations of Santorio 2018, 2020, and with some remarks on the variation in the robustness and cancellation of conjunctive inferences.

2 An asymmetry in scope taking

Disjunction and indefinites can take scope out of islands, while conjunction and universal quantifiers cannot. Consequently, sentences in which this happens lack parallel conjunctive and universal quantifier alternatives, as these are not generated by grammar. This may result in their sets of alternatives not being closed under conjunction, and hence them potentially giving rise to conjunctive inferences. We argue that this possibility is realized.

2.1 Exceptional scope taking and alternatives

Disjunction and indefinites allow for exceptional scope, that is, they can take scope out of islands (e.g., Fodor and Sag 1982, Rooth and Partee 1982). For illustration, the sentence in (13), in which disjunction occurs in the restrictor of a universal quantifier at surface form, can convey merely that one of Gal and Tal is such that everyone who knows them is lucky. The reading can be brought out by the parenthetical "I don't remember who." (Exceptional scope readings may require focal stress on the disjunctive marker or the indefinite determiner, see, e.g., Schlenker 2006, Endriss 2009.)

(13) Everyone who knows Gal or Tal is lucky.
Can convey: (everyone who knows Gal is lucky) $\vee$ (everyone who knows Tal is lucky)

Similarly, the sentence in (14), in which an indefinite replaces the above disjunction, can convey

(ii) Anyone that had time and patience contributed to the Britannica.

merely that there is a student such that everyone who knows them is lucky. In addition to the above parenthetical, this reading can be brought out by a continuation like "namely, Gal."

(14) Everyone who knows a student is lucky.
Can convey: $\exists$x: student x $\wedge$ (everyone who knows student x is lucky)

In contrast, conjunction and universal quantifiers do not allow for such exceptional scope (cf., e.g., Fodor and Sag 1982, Rooth and Partee 1982, May 1985). For illustration, a wide-scope construal of conjunction and a universal quantifier in (15)-(16) would entail, say, that everyone who knows just Gal is lucky, an entailment that the sentence clearly lacks (conversely, it would lead to meanings that are too weak if this sentence were embedded under negation; see, e.g., Ruys 1992, Mayr and Spector 2012, Ruys and Spector 2017 on issues involved in probing parses that yield stronger readings).

(15) Everyone who knows Gal and Tal is lucky.
Cannot convey: (everyone who knows Gal is lucky) $\wedge$ (everyone who knows Tal is lucky)

(16) Everyone who knows every student is lucky.
Cannot convey: $\forall$x: student x $\rightarrow$ (everyone who knows student x is lucky)

The unavailability of exceptional scope of conjunction and universal quantifiers can be brought out more sharply with examples in which the low-scope, but not the exceptional scope, construal of the quantifier is pragmatically infelicitous. One such example is provided in (17) (due to Winter 2002): if the universal quantifier could take exceptional scope, we would get a reasonable meaning that no matter who Gal's mother is, Gal should be fine. The fact that we only get an odd meaning for the sentence indicates that exceptional scope cannot be assigned to universal quantifiers.

(17) %If every woman gave birth to Gal, Gal should be fine.

On the basis of such, and many other arguments, we conclude that a low-scope construal of conjunction and universal quantifiers is the only one available in examples like the above ones – conjunction and universal quantifers cannot take scope out of syntactic islands.

Exceptional movement. Various analyses of exceptional scope readings of disjunction and indefinites have been proposed (e.g., Reinhart 1997, Winter 1997, Kratzer 1998, Matthewson 1999, Schwarzschild 2002, Schlenker 2006, Endriss 2009, Brasoveanu and Farkas 2011, among many oth-

ers). For concreteness, we assume that disjunction and indefinites have their standard quantificational meanings, and that exceptional scope is achieved by quantifier raising (e.g., Geurts 2000, Schwarz 2001, see also Barker 2022 for discussion). On this approach, the sentences in (13)-(14) have the LFs in (18), where the disjunctive and indefinite phrases have moved out of the relative clause.

(18) a. [Gal or Tal] [everyone who knows ~~[Gal or Tal]~~ is lucky]

b. [a student] [everyone who knows ~~[a student]~~ is lucky]

possible exceptional movement

Movement structures are interpreted by having the lower copies of the moved phrases transformed into definite descriptions of objects bound by the moved phrase (Trace Conversion, e.g., Fox 2002, Sauerland 2004). The meaning of the structures in (18) are provided in (19) – they correspond to the target exceptional scope readings of the disjunction and the indefinite.

(19) a. (everyone who knows Gal is lucky) $\vee$ (everyone who knows Tal is lucky)

b. $\exists$x: student x $\rightarrow$ (everyone who knows student x is lucky)

Parallel LFs are unavailable for the corresponding sentences with conjunction and universal quantifiers, as indicated in (20). This holds by fiat on the approach here, but can be derived on principled grounds on more sophisticated movement approaches (esp., Demirok 2019, Ch. 4, on whose proposal anomalous truth conditions are assigned to structures in which non-indefinites take exceptional scope, and Charlow 2020, on whose proposal only nominals that are underlyingly predicative, such as indefinites, can undergo exceptional scope shift).

(20) a. *[Gal and Tal] [everyone who knows ~~[Gal and Tal]~~ is lucky]

b. *[every student] [everyone who knows ~~[every student]~~ is lucky]

impossible movement

It may well be that the simple movement theory of exceptional scope is overly simple. We believe, however, that the choice of the theory is not crucial for the purposes of the paper, and that the convenience of its simplicity outweighs its familiar shortcomings here. More to the point, any theory that (i) assigns exceptional scope to the existential quantification introduced by disjunction and indefinites, or something akin to it, and (ii) does not assign something similar to conjunction and universal quantifiers should do for our purposes. Hence, the choice function approaches with

existential closure (e.g., Reinhart 1997, Winter 1997, Matthewson 1999) would do as well. In order to lend some credence to this belief, we discuss alternative approaches at different points below.

Missing and available alternatives. A consequence of the definition of alternatives in (5) is that sentences in which disjunction and indefinites take exceptional scope lack parallel alternatives in which these expressions are replaced by conjunction and universal quantifiers, as stated in (21) – namely, grammar cannot generate them (but see Charlow 2019 for an alternative assumption).

(21) a. *[Gal and Tal] [everyone who knows ~~[Gal and Tal]~~ is lucky]
$\notin$ ALT([Gal or Tal] [everyone who knows ~~[Gal or Tal]~~ is lucky])

b. *[every student] [everyone who knows ~~[every student]~~ is lucky]
$\notin$ ALT([a student] [everyone who knows ~~[a student]~~ is lucky])

But this does not mean that such sentences lack any alternatives whatsoever. Which ones they induce is again conditioned by the constraint in (5), that is, by what the complexity limitations admit and by what grammar can generate. Given our representation of movement, we expect the disjunctive sentence in (18a) to have the alternatives in (22): the low-scope disjunct alternatives (rows 1-2), the low-scope disjunctive alternative (row 3), and the low-scope conjunctive alternative (row 4).

(22) [everyone who knows Gal is lucky],
[everyone who knows Tal is lucky],
[everyone who knows [Gal or Tal] is lucky],
[everyone who knows [Gal and Tal] is lucky]
$\in$ ALT([Gal or Tal] [everyone who knows ~~[Gal or Tal]~~ is lucky])

The indefinite sentence in (18b) has the alternatives provided in (23): it has the simple subdomain alternatives (row 2), the low-scope existential quantifier subdomain alternatives (row 3), and the low-scope universal quantifier subdomain alternatives (row 4). If Gal and Tal are the only relevant students, the subdomain alternatives correspond to the disjunct alternatives in (22), and the low-scope quantifier alternatives correspond to the low-scope disjunctive and conjunctive alternatives.

(23) For any D′, where $[\![\text{D'}]\!] \subseteq [\![\text{D}]\!]$,
[$a_{D'}$ student] [everyone who knows ~~[$a_{D'}$ student]~~ is lucky],
[everyone who knows [$a_{D'}$ student] is lucky],

[everyone who knows [every$_{D'}$ student] is lucky]

$\in$ ALT([a$_D$ student] [everyone who knows ~~[a$_D$ student]~~ is lucky])

In short, we expect the surface forms of the initial sentences, on which disjunction and indefinites are interpreted in the relative clause, to be alternatives to the initial sentences. These alternatives will not play an important role in much of the paper, but we substantiate their assumption in the following by showing that they correctly inhibit potential conjunctive strengthenings as well as that they allow us to derive the observed strengthened readings of the sentences we just discussed.

Inhibition. The absence of parallel conjunctive and universal quantifier alternatives may have considerable repercussions for what inferences disjunctive and indefinite sentences can give rise to. Missing a parallel conjunctive or universal quantifier alternative is not enough by itself, however, to warrant conjunctive strengthening. This is the case for the sentences in (13)-(14) discussed above – even though their exceptional scope structures lack parallel conjunctive and universal quantifier alternatives, their sets of alternatives are closed under conjunction. For example, this holds for the set of alternatives to the sentence (13), provided in (22) above: in particular, the conjunction of the disjunct alternatives is equivalent to the low-scope disjunctive alternative.[4] Accordingly, conjunctive strengthening of the sentence is predicted not to be possible.

(24) **Condition (C) precludes potential conjunctive inference for (13):**

ALT([Gal or Tal] [everyone who knows ~~[Gal or Tal]~~ is lucky]) is closed under conjunction.

$\big($esp. $[\![$[everyone who know Gal is lucky]$]\!] \wedge [\![$[everyone who knows Tal is lucky]$]\!]$

$\Leftrightarrow [\![$[everyone who knows Gal or Tal is lucky]$]\!]\big)$

In fact, on its exceptional scope parse, the sentence in (13) gives rise to a negated conjunctive inference, namely, that it is not the case that both Gal and Tal are such that everyone who knows them is lucky (as discussed by Charlow 2019 in his pioneering work on scalar implicatures of exceptional

[4]This equivalence holds on the assumption that either universal quantifiers do not induce existential presuppositions that their restrictors are non-empty (these would then have to arise as implicatures, say) or that these are factored out in the application of condition (C). With respect to the latter option, it is worth noting that a felicitous assertion of the exceptional scope disjunctive sentence in (13) requires the existential presuppositions of both disjuncts to be satisfied in the context (cf. Karttunen 1974). Now, in any such context, the conjunction of the disjunct alternatives and the low-scope disjunctive alternative are contextually equivalent. As will become clear in our actual computations of conjunctive strengthening, such contextual equivalence suffices for blocking conjunctive strengthening.

scope elements; see Sect. 5.1 for further discussion). This inference can be derived precisely by negating the low-scope disjunctive alternative.[5] The same reasoning and conclusions extend to the indefinite sentence in (14).[6] We can thus take our representations and the alternatives they induce to be supported, both by their blocking of undesirable inferences and by their enabling the derivation of observed inferences.[7,8]

[5]The inference is derived by means of exhaustification, which we introduce in the following section. Following Fox 2007, we implement it by attaching an exhaustification operator *exh* at matrix level of the sentence, which then negates the low-scope disjunctive alternative in the case at hand (the low-scope disjunctive alternative is excludable, while the disjunct and the low-scope conjunctive alternatives are not). See Sect. 2.2 for the definitions and details.

[6]If disjunction has more than two disjuncts, or if the domain of the indefinite in (14) consists of more than two students, several readings can be derived for the sentence, all else equal. The readings vary with respect to which of the alternatives to the sentences are relevant in the context. For example, focusing on the indefinite sentence, if no proper subdomain alternatives are relevant, that is, if only the low-scope existential quantifier alternatives with the same domain as the initial indefinite is relevant, the reading that is derived for (14) is that not every student is such that everyone who knows them is lucky. On the other hand, if all alternatives are taken to be relevant, the reading that is derived for (14) is that exactly one student is such that everyone who knows them is lucky. Charlow 2019 notes that these are indeed available readings for the sentence. A reviewer points out, however, that further, intermediate readings may be expected to obtain in appropriate contexts, that is, readings that are properly entailed by the latter reading and that properly entail the former reading. One such reading may be that (exactly) one or two students are such that everyone who knows them is lucky (cf. Bar-Lev 2018 for a related discussion of 'non-maximal' readings of definite plurals). Whether such readings are indeed available, and whether they can be derived by the theory as independent readings in the way hinted at, are challenging tasks that will have to be pursued elsewhere. Their pursuit should proceed in concert with a development of a theory of constraints on the pruning of (subdomain) alternatives (cf. Crnič et al. 2015, Bar-Lev 2018). See also fn. 10.

[7]A reviewer notes that the sentence in (i), in which a universal quantifier is topicalized across a modified numeral indefinite in the subject position, conveys on neutral intonation that it is false that more than two female linguists read every book. This inference can be derived by negating the low-scope universal quantifier alternative to the sentence, provided in (iib), tentatively lending further support to our assumption of low-scope alternatives. A host of further predictions are generated on the assumption of low-scope alternatives, predictions that we cannot explore further here.

(i) Jedes Buch haben mehr als zwei Linguistinnen gelesen.
every book have more than 2 linguists.F read
"Every book is such that more than two female linguists read it."

(ii) a. LF: [every book] [[more than 2 linguists] read ~~[every book]~~]
b. Alternative: [[more than 2 linguists] read [every book]]

[8]Meyer and Sauerland 2009 discuss detectability of inverse scope readings of various sentences. They propose that sentences that have stronger inverse scope readings than surface scope readings cannot be judged as false in situations

Avoiding inhibition? We have thus not shown yet that exceptional scope asymmetries affect strengthening in ways conducive to generating conjunctive inferences. Are there cases in which low-scope alternatives to exceptional scope structures do not inhibit conjunctive strengthening? We have just seen that to find these we have to look for (i) environments that are not (Strawson) anti-additive (where the conjunctive inference would be equivalent to the low-scope disjunctive/existential quantifier alternative). Furthermore, (ii) the environments cannot be distributive (where the conjunctive inference would be be equivalent to the low-scope conjunctive/universal quantifier alternative). Finally, (iii) if disjunction or an indefinite occur in an upward-monotone environment, conjunctive strengthening of exceptional scope elements might be difficult to distinguish from conjunctive strengthening of the sentences on their surface form construal (recall our discussion of sentence (1) above). One class of cases that fall within these boundary conditions are sentences in which disjunction and indefinites surface in restrictors of proportional quantifiers.

2.2 Supporting evidence

Consider the sentence in (25), in which disjunction occurs in the restrictor of *most*, a non-monotone environment (see Bar-Lev and Fox 2020, fn. 46), as well as the sentence in (26), in which disjunction is replaced with its nominal cousin, an *either* phrase. (As suggested above, the sentences may have to be read with focal stress on the disjunctive marker and the indefinite determiner in order to obtain the target readings we discuss below.)

(25) Most kids that are on team A or team B are on both teams.

in which their surface scope readings are true but their inverse scope readings are false due to 'truth dominance': if a sentence on its more accessible reading (that is, surface scope) reading is judged as true in a situation, the sentence must be judged as true. A reviewer points out that computation of scalar implicatures on the basis of low-scope alternatives may affect the application of this principle as it allows for breaking entailment between surface and inverse scope readings. In fact, it allows the readings to be complmentary: if one computes the strenghtening inference for the surface scope construal of a sentence, the sentence's inverse scope reading will be incompatible with it. Does this affect the predictions of the truth dominance principle? Not obviously. Consider a situation in which the inverse scope reading is true, but the strenghtened surface scope reading is not – this is a situation in which the non-strengthened surface scope reading is true as well, so judging the sentence as true may well be due to assigning the sentence a non-strengthened surface scope parse. On the other hand, consider a situation in which the inverse scope reading is false – this is a situation in which the strenghtened surface scope reading is true (unless none of the readings is true, making the situation irrelevant). Truth dominance predicts the sentence to be evaluated as true in this case (as the surface scope reading is the more accessible reading), just as in the case of the non-strengthened construal of the sentence.

(26) Most kids that are on either team are on both teams.

Conjunctive and missing disjunctive meaning. The sentences can correctly describe a situation in which there are two teams, team A and team B, each with 5 kids as members, and with 3 kids being on both teams (that is, 4 of the 7 kids are on a single team, 3 of the 7 kids are on both teams). Given this setup, the sentences would be false if disjunction and *either* would contribute their usual meanings in their surface position, provided in (27a) (only 3 of the 7 kids are on both teams). The sentences are true if the conjunctive inference is computed, provided in (27b), as indeed each team is such that the majority of kids on it that are also on the other team (both times 3 of the 5 kids).

(27) **(Un)observed inferences of sentences (25)-(26):**

a. $\not\Rightarrow$ [[most]]({x | kid x on team A or B})({x | kid x on both teams})

b. $\Rightarrow$ [[most]]({x | kid x on team A})({x | kid x on both teams}) $\wedge$
[[most]]({x | kid x on team B})({x | kid x on both teams})

That the sentences in (25)-(26) can indeed convey the conjunctive inference in (27b), rather than merely an exceptional scope disjunctive one, is intuitively obvious. For example, when asked about either team, if one accepts the sentences in (25)-(26), one can answer that most of its members are on both teams. Moreover, we judge the sentences as false in a slightly modified scenario in which team B has 10 kids, but everything else stays the same: namely, in this scenario, an exceptional scope disjunctive meaning is true (because most kids on team A are on both teams), but the conjunctive inference is false (because it is false that most kids on team B are on both teams).[9] (See Sect. 5.2 for a further discussion of the readings of such sentences on their exceptional scope construal.)

In the following, our discussion tracks the entailment patterns described in (27). The first pattern pertains to the scope of disjunction and indefinites: they do not contribute their meaning in their surface positions, but at the matrix level. However, what we observe at the matrix level are conjunctive meanings, not merely disjunctive or existential ones. This is captured by exhaustification.

[9]Similar judgments are available for sentences with other proportional quantifiers, such as those provided in (i), in apropriately modified scenarios. Our account of the pattern in the main text extends to these data in obvious ways:

(i) a. More than a third of the kids that are on team A or team B are on both teams.

b. Exactly one third of the kids that are on team A or team B are on both teams.

c. 60% of the kids that are on team A or team B are on both teams.

Scope. Disjunction and the *either* NP can evade contributing their meaning in their surface scope positions by taking exceptional scope. Specifically, the sentences in (25)-(26) can be assigned the exceptional scope structures in (28), which have the meaning in (29).

(28) a. [team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams]
b. [either team] [most kids that are on ~~[either team]~~ are on both teams]

(29) $[\![\text{most}]\!](\{x \mid \text{kid x is on team A}\})(\{x \mid \text{kid x is on team A and team B}\}) \vee$
$[\![\text{most}]\!](\{x \mid \text{kid x is on team B}\})(\{x \mid \text{kid x is on team A and team B}\})$

The meaning in (29) is clearly independent of the low-scope disjunctive one – for example, it may be that most kids on team A are on both teams, but constitute only a small minority of team B. Yet, the exceptional scope disjunctive meaning in (29) is not all there is to the meaning of (25)-(26) – it is not strong enough (namely, the sentences give rise to a conjunctive inference). In our derivation of this inference, we focus on the disjunctive sentence, not least because the conjunctive strengthening of the indefinite sentence proceeds analogously.

Strengthening. The structure in (28a) lacks a parallel conjunctive quantifier alternative since this is not generated by grammar, as stated in (30). It has the alternatives in (31): the initial sentence, a low-scope disjunctive alternative, two disjunct alternatives, and a low-scope conjunctive alternative.

(30) *[team A and team B] [most kids that are on ~~[team A and team B]~~ are on both teams]
$\notin$ ALT([tA or tB] [most kids that are on ~~[team A or team B]~~ are on both teams])

(31) ALT([team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams]) =

{ [team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams]
[most kids that are on [team A or team B] are on both teams]
[most kids that are on team A are on both teams]
[most kids that are on team B are on both teams]
[most kids that are on [team A and team B] are on both teams] }

The set of alternatives in (31) is not closed under conjunction: in particular, the conjunction of the disjunct alternatives is not equivalent to the low-scope disjunctive alternatives (as witnessed by our initial scenario on which the former is true but the latter is false) or any other alternative for that matter. Accordingly, conjunctive strengthening may be possible.

(32) **Condition (C) admits potential conjunctive inference for (25):**

ALT([team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams])

is not closed under conjunction.

Conjunctive strengthening is effected by exhaustification, which is encoded in operator *exh*. *Exh* can attach at any clausal level, and it negates all relevant excludable alternatives, as defined in (33)-(34) (the version in Katzir 2014, Crnič et al. 2015, a slight revision of Fox 2007). We represent excludable alternatives as semantic objects, as interpretations of elements that are in all maximal subsets of formal alternatives to a sentence whose joint negation is consistent with the sentence.

(33) **Exhaustification:**

$[\![\text{exh}_C\ S]\!] = [\![S]\!] \wedge \forall p \in \text{IE}(S) \cap C\text{: } \neg p$

(34) **Excludable alternatives:**

$\text{IE}(S) = \bigcap \{M' \mid$ for a maximal subset M of ALT(S) such that $\{\neg [\![S']\!] \mid S' \in M\} \cup \{[\![S]\!]\}$ is consistent, M' is the set of meanings of the elements of $M\}$

With these definitions in hand, we can turn to the sentence in (25). The full parse of the sentence that yields the conjunctive inference involves recursive exhaustification, as provided in (35):

(35) $[\text{exh}_{C'}$ [exh_C [teamA or teamB] [most kids that are on ~~[teamA or teamB]~~ are on both teams]]]

At the first layer of exhaustification, the low-scope disjunctive alternative is excludable, as provided in (36): it can hold that either most students on team A or most kids on team B are on both teams, while it is false that most of all the kids are on both teams (recall the scenario discussed above in which the majority of team A is also on team B, but team B is huge, hence team A kids make up only a minority of the kids on it). Since the conjunctive alternative is a tautology, and since the disjunct alternatives are symmetric, none of these other alternatives are excludable.

(36) IE([team A or team B] [most kids that are on ~~[team A or team B]~~ are on both teams]) =

$\{ [\![\text{most}]\!](\{x \mid$ kid x is on team A or B$\})(\{x \mid$ kid x is on team A and team B$\})\ \}$

The output of the first layer of exhaustification on the assumption that the low-scope disjunctive alternative is not relevant is provided in (37) (we assume throughout that non-disjunct alternatives

are not relevant in order to simplify our derivations[10]): it corresponds to the meaning of the sister of *exh*, that most kids on team A or most kids on team B are on both teams.

(37) $[\![[\text{exh}_C$ [teamA or teamB] [most kids that are on ~~[teamA or teamB]~~ are on both teams]]$]\!]$ =

$\big([\![\text{most}]\!](\{x \mid \text{kid x is on team A}\})(\{x \mid \text{kid x is on team A and team B}\}) \vee$

$[\![\text{most}]\!](\{x \mid \text{kid x is on team B}\})(\{x \mid \text{kid x is on team A and team B}\})\big)$

At the second layer of exhaustification, the disjunct alternatives become excludable, as provided in (38), which is witnessed by their joint exclusion being compatible with the sentence being true. The output of the matrix exhaustification on our standing assumption that only disjunct alternatives are relevant is computed in (39). Specifically, exhaustification generates for each team the inference that if most kids on it are on both teams, then the same holds for the other team. And since this does hold for one team, as computed in (37), it must hold for both teams.

(38) IE([exh$_C$ [teamA or teamB] [most kids that are on ~~[teamA or teamB]~~ are on both teams]]) =

$\{\ [\![\text{most}]\!](\{x \mid \text{kid x is on team A}\})(\{x \mid \text{kid x is on team A and team B}\}) \wedge$

$\neg[\![\text{most}]\!](\{x \mid \text{kid x is on team B}\})(\{x \mid \text{kid x is on team A and team B}\})$

$[\![\text{most}]\!](\{x \mid \text{kid x on team B}\})(\{x \mid \text{kid x is on team A and team B}\} \wedge$

$\neg[\![\text{most}]\!](\{x \mid \text{kid x is on team A}\})(\{x \mid \text{kid x is on team A and team B}\}$

$[\![\text{most}]\!](\{x \mid \text{kid x is on team A or B}\})(\{x \mid \text{kid x is on team A and team B}\} \wedge$

$\neg[\![\text{most}]\!](\{x \mid \text{kid x is on team A}\})(\{x \mid \text{kid x is on team A and team B}\} \wedge$

$\neg[\![\text{most}]\!](\{x \mid \text{kid x is on team B}\})(\{x \mid \text{kid x is on team A and team B}\}\ \}$

(39) $[\![[\text{exh}_{C'}$ [exh$_C$ [tmA or tmB] [most kids that are on ~~[tmA or tmB]~~ are on both teams]]]$]\!]$ =

$[\![\text{most}]\!](\{x \mid \text{kid x is on team A}\})(\{x \mid \text{kid x is on team A and team B}\} \wedge$

$[\![\text{most}]\!](\{x \mid \text{kid x is on team B}\})(\{x \mid \text{kid x is on team A and team B}\}$

The meaning in (39) corresponds to the conjunctive inference that most kids on team A are on

[10] In many cases, if additional alternatives are taken to be relevant (= not pruned), additional inferences may be generated. In the case at hand, the additional inference would be that it is false that most of all the kids are on both teams. In the one case below in which pruning is necessary to generate a conjunctive inference, this fact is discussed explicitly (see fn. 23). Finally, where (exhaustified) disjunct alternatives are excludable, their pruning is constrained by a principle that requires their joint resolution (cf. Bar-Lev and Fox 2020 on 'cell identification'). See Gotzner and Romoli 2022 for a review of data that suggest ease of pruning of non-disjunct alternatives, in contrast to disjunct alternatives.

both teams, and most kids on team B are on both teams. We thus showed that this inference follows naturally on the assumptions set out above. First: the lack of a low-scope disjunctive contribution of disjunction follows from it being assigned exceptional scope. In fact, the low-scope disjunctive meaning can be negated by exhaustification. Second: the sentence on this exceptional scope parse satisfies condition (C), and exhaustification strengthens its meaning to a conjunctive one.[11]

2.3 Choice functions and alternatives

We conjectured that our choice of the theory of exceptional scope was not critical. We provide some support for this claim by discussing an alternative (the discussion can be skipped without loss of continuity, though see Sect. 4.3). A prominent alternative treatment of exceptional scope of disjunction and indefinites takes these to introduce choice functions that are existentially closed, as defined in (40) (e.g., Reinhart 1997, Winter 1997, 2002, Matthewson 1999, Schlenker 2006).

(40) **Choice functions and existential closure:**
Let E be a non-empty set of individuals. A function f: $\mathscr{P}$(E) $\rightarrow$ E is a choice function, f $\in$ CH, iff for every A $\subseteq$ E: if A is not empty then f(A) $\in$ A. Choice function variables can be bound by an existential closure operator $\exists$ that can attach at any clausal level.

On this theory, sentences (13)-(14) may be assigned the structures in (41a)-(42a): matrix existential closure binds the choice function variables introduced by disjunction and the indefinite; the choice functions' arguments are the set of individuals denoted by the disjoined expressions and the domain of the indefinite. The meanings of the structures correspond to the exceptional scope readings of disjunction and the indefinite, as indicated in (41b)-(42b).

(41) a. [$\exists_f$ [everyone who knows [Gal or$_f$ Tal] is lucky]
b. $\exists$f$\in$CH: (everyone who knows f({Gal, Tal}) is lucky)
$\Leftrightarrow$ (everyone who knows Gal is lucky) $\vee$ (everyone who knows Tal is lucky)

[11] It is worth noting that the parse that gives rise to the target inference in the main text is more complex than those that do not (the former involves, at least, one additional application of scope shift). Accordingly, it should be less accessible and preferred than other parses of the sentence. Although the speakers that we consulted about the data accessed the target reading on appropriate intonation, here and for the other examples in the paper, other disambiguations were often initially preferred. Moreover, some of the speakers accessed the target readings more easily with indefinite sentences. The same holds for the sentences in other languages whose speakers we consulted (Slovenian, German, Arabic, Hebrew).

(42) a. [$\exists_f$ [everyone who knows [a$_f$ student] is lucky]

b. $\exists$f$\in$CH: (everyone who knows f({x | x student}) is lucky)

$\Leftrightarrow \exists$x: student x $\wedge$ (everyone who knows student x is lucky)

In line with our above assumptions, the structures in (41)-(42) lack parallel conjunctive and universal quantifier alternatives since grammar cannot generate them, either due to an independent constraint on their movement or the lexicon not containing a universal closure operator (but see Charlow 2019 for an alternative assumption):

(43) *[Gal and Tal] [everyone who knows ~~[Gal and Tal]~~ is lucky],

*[$\forall_f$ [everyone who knows [Gal or$_f$ Tal] is lucky]]

$\notin$ ALT([$\exists_f$ [everyone who knows [Gal or$_f$ Tal] is lucky])

(44) *[every student] [everyone who knows ~~[every student]~~ is lucky],

*[$\forall_f$ [everyone who knows [a$_f$ student] is lucky]]

$\notin$ ALT([$\exists_f$ [everyone who knows [a$_f$ student] is lucky])

Do the choice function representations admit low-scope connective and quantifier alternatives? Given the theory, the low-scope conjunctive and universal quantifier alternatives are clearly admitted: the existential closure operator can be removed, and the conjunctive marker can replace the disjunctive one, (45), or the universal quantifier can replace the indefinite, (46).

(45) [everyone who knows [Gal and Tal] is lucky]

$\in$ ALT([$\exists_f$ [everyone who knows [Gal or$_f$ Tal] is lucky])

(46) [everyone who knows [every student] is lucky]

$\in$ ALT([$\exists_f$ [everyone who knows [a$_f$ student] is lucky])

The structures in (41a)-(42a) plausibly have low-scope disjunctive or existential alternatives as well, as stated in (47)-(48). The source of these alternatives depends on the details of the theory, however. It may either follow from disjunction and indefinites being ambiguous and allowing for both standard and choice function construals (cf., e.g., Reinhart 1997, Kratzer 1998, Matthewson 1999, but not Winter 1997, 2002; see also Karttunen 1968, Fodor and Sag 1982 for related ambiguity proposals), or by taking existential closure to (potentially vacuously) apply at each clausal level, as suggested by a reviewer (or by taking existential closure to apply in tandem with compo-

sitional interpretation).[12] In any case, it seems reasonable to contend that low-scope disjunctive and existential quantifier alternatives are generated by grammar.

(47) [everyone [who knows [Gal or Tal]] is lucky], or
[everyone [$\exists_f$ who knows [Gal or$_f$ Tal]] is lucky]
$\in$ ALT([$\exists_f$ [everyone who knows [f Gal or Tal] is lucky])

(48) [everyone [who knows [a student]] is lucky], or
[everyone [$\exists_f$ who knows [a$_f$ student]] is lucky]
$\in$ ALT([$\exists_f$ [everyone who knows [a$_f$ student] is lucky])

An adoption of a choice function approach to disjunction and indefinites thus does not seem to differ from the simple movement approach in any respect pertinent to the goals of this paper at this point: they both involve structures in which disjunction and indefinites, or correspondingly existential quantification over disjuncts, apply at the level of exceptional scope; they both predict that there are no exceptional scope conjunction and universal quantifier alternatives; and they both allow for the low-scope connective and quantifier alternatives. Accordingly, for reasons of simplicity, we stick to the simple movement approach in the following (but see Sect. 4.3 for further discussion).

3 Another asymmetry in scope taking

Another difference in the behavior of disjunction and indefinites vs. conjunction and universal quantifiers is that the former, but not the latter, can take scope out of certain downward-monotone environments. Consequently, sentences in which this happens lack parallel conjunctive and universal

[12] The existence of low-scope alternatives may also follow naturally from a theory of generation of alternatives, which is usually left implicit in the discussion of the theory of alternatives. One candidate for such a theory, which would have condition (5) as a consequence, would take alternatives to a sentence to be derived from computational workspaces related in a specific way to that of the initial sentence (see Chomsky 1995 on background assumptions): these workspaces could consists of all the material used in constructing the initial sentence, or a proper subset of the initial workspace, or some elements of the initial workspace could be replaced with their alternatives, or some mixture of these. Note that on this characterization, no stipulation need be made at which level existential closure operation can be attached in the alternatives. The theory would also admit the same alternatives on the movement approach to exceptional scope adopted in the preceding subsection: for example, a low-scope disjunctive alternative of the exceptional scope disjunctive sentence could be derived either from the same workspace as the latter (if there is no morpheme triggering exceptional movement) or the same workspace minus the functional exceptional movement trigger.

quantifier alternatives. Their sets of alternatives might then not be closed under conjunction, and hence they may give rise to conjunctive inferences. We argue that this possibility is realized.

3.1 Scoping out of downward-monotone environments

Disjunction and indefinites can scope out of certain downward-monotone environments, while conjunction and universal quantifiers cannot. These observations have been discussed extensively in recent literature in relation to nominal quantifiers (see, e.g., Mayr and Spector 2012, Fleisher 2015; see also Liu 1990, Beghelli 1995 for related earlier discussion). For example, disjunction and indefinite may take exceptional scope above downward-monotone operators like *fewer than 1000 students* and Strawson downward-monotone operators like *only 900 students* (that is, operators that are downward-monotone once their presuppositions are factored out, von Fintel 1999), as exemplified in (49)-(50), where the wide-scope reading of disjunction and the indefinite is forced by the continuation "I don't remember which."

(49) a. Fewer than 1000 students got into Cambridge or Oxfor. I don't remember which.

b. Fewer than 1000 students got into a UK university. I don't remember which.

Reading: ✓ or/a $>$ fewer than 1000

(50) a. Only 900 students got into Cambridge or Oxford. I don't remember which.

b. Only 900 students got into a UK university. I don't remember which.

Reading: ✓ or/a $>$ only 900

This scopal behavior is not replicated with conjunction and universal quantifiers, as exemplified in (51)-(52) (esp., Fleisher 2015, fn.25, for the behavior of universal quantifiers under modified numeral indefinites). The sentences in (51)-(52) thus do not have parses on which conjunction and the universal quantifier would take scope over the (Strawson) downward-monotone operators.[13]

[13] A reviewer asks about the readings of (49) if we substitute *each* for *every* . While the discussion in the literature focused on *every* (perhaps also because *each* NPs seem to be marked in certain downward-monotone environments, cf. Beghelli and Stowell 1997), it may well be that the scopal behavior of *each* is more flexible, at least for some speakers. If this is indeed the case, and if the speakers that admit this extra flexibility nonetheless compute conjunctive inferences for the examples at hand (the reviewer is one of them), then the non-inhibition by *each* could be due to it not being an alternative to the indefinite determiner (as the reviewer notes). This may well align with the claim that *each* behaves like a high distributive head rather than a usual determiner head (e.g., Sportiche 1988, Beghelli 1995, Beghelli and Stowell 1997, among others). The reviewer also raises a related question about (complex) conjunctions. In this case, too,

(51) a. Fewer than 1000 students got into Cambridge and Oxford.

b. Fewer than 1000 students got into every UK university.

Readings: ✓ fewer > and/every; ✗ and/every > fewer

(52) a. Only 900 students got into Cambridge or Oxford.

b. Only 900 students got into every UK university.

Readings: ✓ only > and/every; ✗ and/every > only

A comprehensive description of this behavior as well as its explanation is still outstanding, which is similar to what holds for island constraints on movement. One promising approach takes the unavailability of wide-scope for conjuntion in these configurations to follow from a condition on quantifier raising that requires it to weaken the meanings of the pertinent structures (see Mayr and Spector 2012 and Fleisher 2015). What matters here is not the source of the restriction on scope, rather merely the fact that some disjunctive and indefinite sentences on the wide-scope construal of disjunction and indefinites lack parallel conjunctive and universal quantifier alternatives, as stated in (53)-(54) – again, this absence follows from grammar not generating them.

(53) *[Cambridge and Oxford] [fewer than 1000 students got into ~~[Cambridge and Oxford]~~]
$\notin$ ALT([Cambridge or Oxford] [fewer than 1000 students got into ~~[Cambridge or Oxford]~~])

(54) *[every UK university] [fewer than 1000 students got into ~~[every UK university]~~]
$\notin$ ALT([a UK university] [fewer than 1000 students got into ~~[a UK university]~~])

This state of affairs is consequential.

3.2 Supporting evidence

Consider the sentence in (55a), in which disjunction surfaces in the scope of a downward-monotone quantifier *fewer than 1000 students*, as well as the sentence in (55b), in which disjunction is replaced

if conjunctive phrases could take scope outside of downward-monotone environments, and would count as alternatives to disjunctive phrases, this should inhibit conjunctive strengthening. To the extent speakers admit wide-scope construal for conjunction in the examples at hand, but nonetheless compute conjunctive inferences, the conjunctive phrases to which they can assign wide scope would have to not count as alternatives to disjunction (see, e.g., Dočekal et al. 2022 on conjunctive phrase structures of differing complexity). In light of Sect. 2.3, one source of this may be that the wide-scope disjunctive sentences have a choice function parse (cf. Mayr and Spector 2012) that lacks wide-scope quantifier raising alternatives. We have to leave a systematic study of these issues to a future occassion.

by its nominal cousin, an *either* phrase. Now, let's assume, more than 1000 students get into Oxford or Cambridge each year, though Oxford and Cambridge each accepts fewer than 1000 students. This state of affairs can be described by the sentences in (55). (Again, the disjunctive marker and the indefinite determiner may need to be stressed to access this reading.)

(55) a. Fewer than 1000 students got into Cambridge or Oxford.

b. Fewer than 1000 students got into either university.

This is unexpected on the surface scope construal of the sentence, whose meaning is provided in (56a), since it is false in the above scenario – more than 1000 students got into one of these universities. Moreover, the sentences give rise to the conjunctive inference provided in (56b), so simple wide-scope disjunctive and indefinite construals of the sentences do not suffice to capture the sentences' meanings on their own.

(56) **(Un)observed inferences of sentences in (55):**

a. $\not\Rightarrow$ card({x | student x got into Cambridge or Oxford})<1000

b. $\Rightarrow$ card({x | student x got into Cambridge})<1000 $\wedge$
card({x | student x got into Oxford})<1000

A similar state of affairs obtains also with occurrences of disjunction and the *either* phrase in the scope of Strawson downward-monotone operators like the *only* NP in (57). We can judge the sentences as good descriptions of the scenario in which exactly 900 students got into Cambridge and exactly 900 other students got into Oxford.

(57) a. Only 900 students got into Cambridge or Oxford.

b. Only 900 students got into either university.

The fact that we can judge the sentences as true in the above scenario indicates that they do not convey a low-scope disjunctive or existential meaning, provided in (58a), as more than 900 students got into Cambridge or Oxford. They also gives rise to the conjunctive inference in (58b).

(58) **(Un)observed inferences of sentences in (57):**

a. $\not\Rightarrow$ card({x | student x got into Cambridge or Oxford}) = 900

b. $\Rightarrow$ card({x | student x got into Cambridge}) = 900 $\wedge$

$$\text{card}(\{x \mid \text{student x got into Oxford}\}) = 900$$

These patterns match those discussed in the preceding section: disjunction and indefinites do not contribute a disjunctive/existential meaning in the environment in which they appear at surface form, rather they convey a wide-scope conjunctive meaning. Accordingly, the derivation of these patterns can proceed in parallel to those in the preceding section.

Scope and strengthening. We focus on the disjunctive sentence in (55) in the following, with the derivation being straightforwardly extendable to (57). The sentence can be parsed with disjunction taking matrix scope, as provided in (59). But the wide-scope disjunctive meaning, provided in (60), is on its own weaker than the reading we have to derive for the sentence.

(59) [Cambridge or Oxford] [fewer than 1000 students got into ~~[Cambridge or Oxford]~~]

(60) $\text{card}(\{x \mid \text{student x got into Cam.}\}) = 900 \vee \text{card}(\{x \mid \text{student x got into Oxford}\}) = 900$

The meaning of the structure in (59) can be strengthened to a conjunctive one. The set of alternatives to the structure is provided in (61), which crucially lacks a wide-scope conjunctive alternative.

(61) ALT([Cambridge or Oxford] [fewer than 1000 students got into ~~[Cambridge or Oxford]~~]) =

{[Cambridge or Oxford] [fewer than 1000 students got into ~~[Cambridge or Oxford]~~],
[fewer than 1000 students got into [Cambridge or Oxford]],
[fewer than 1000 students got into Cambridge],
[fewer than 1000 students got into Oxford],
[fewer than 1000 students got into [Cambridge and Oxford]]}

The set in (61) is not closed under conjunction, as stated in (62). In particular, the conjunction of the disjunct alternatives is not equivalent to the low-scope disjunctive alternative, as witnessed by the conjunction of the disjunct alternatives being true in the above scenario and the low-scope disjunctive alternative being false. Hence, conjunctive strengthening of the sentence may be possible.

(62) **Condition (C) admits potential conjunctive inference for (55):**
ALT([Cam or Oxford] [fewer than 1000 students got into ~~[Cam or Oxford]~~]])
is not closed under conjunction.

At the first layer of exhaustification, the only excludable alternative to the base sentence is the low-scope disjunctive one. At the second layer of exhaustification, all the alternatives, including the disjunct alternatives, are excludable, as provided in (63). (Recall that we are treating only the disjunct alternatives as relevant, that is, in the domains C and C'.)

(63) IE([exh$_C$ [Cam. or Oxford] [fewer than 1000 students got into ~~Cam. or Oxford~~]]) =

{card({x | student x got into Cambridge})<1000 ∧
¬(card({x | student x got into Oxford})<1000),
card({x | student x got into Oxford})<1000 ∧
¬(card({x | student x got into Cambridge})<1000),
card({x | student x got into Cambridge or Oxford})<1000,
card({x | student x got into Cambridge and Oxford})<1000 ∧
¬(card({x | student x got into Oxford})<1000) ∧
¬(card({x | student x got into Oxford})<1000)}

The recursively exhaustified meaning of the sentence is provided in (64): in particular, negating the disjunct alternatives gives us the proposition that one disjunct alternative is true iff the other one is true as well, and since the sentence entails that at least one of them is true, both must be. Accordingly, the meaning we derived corresponds to the conjunctive inference of the sentence.

(64) ⟦[exh$_{C'}$ [exh$_C$ [Cam. or Oxford] [fewer than 1000 students got into ~~Cam. or Oxford~~]]]⟧ =

card({x | student x got into Cambridge})<1000 ∧
card({x | student x got into Oxford})<1000

In conclusion, we showed that scope and strengthening can conspire to yield conjunctive inferences also when disjunction and indefinites occur at surface form in downward-monotone environments. This was possible because sentences in which disjunction and indefinites take scope out of these environments lack parallel conjunctive and universal quantifier alternatives, and the conjunctions of their disjunct alternatives are not equivalent to any of their other alternatives. The derivation of conjunctive meanings of sentences with *only* NP proceeds analogously.

3.3 Scope and strengthening in NPI licensing

Indefinites that we used to illustrate conjunctive strengthening were *either* NPs. These have a narrower distribution than regular indefinites. For example, they are unacceptable in simple episodic sentences, while they are acceptable in downward-monotone and modal environments.

(65) #Gal got into either university.

(66) a. Gal didn't get into either university.

b. Gal may/must have got into either university.

Moreover, in appropriately modified scenarios, *either* NPs can be replaced in all the above examples with *any* NPs without affecting the acceptability of the sentences or the kinds of inferences the sentences give rise to.[14] For example, consider the scenario in which each Ivy League school accepts fewer than 1000 students, but they jointly accept more than 1000 students. This state of affairs can be felicitously described with the sentence in (67). In short, the NPI does not contribute a low-scope existential meaning, but does give rise to a conjunctive inference, as stated in (68).

(67) Fewer than 1000 students got into any Ivy League school.

(68) **(Un)observed inferences of sentence (67):**

a. $\not\Rightarrow \text{card}(\{x \mid \text{student x got into an Ivy Leauge school}\}) < 1000$

b. $\Rightarrow \bigwedge_{x \in [\![\text{Ivy Leaguge school}]\!]} \text{card}(\{x \mid \text{student x got into x}\}) < 1000$

The derivation of the meaning of the sentence in (67) proceeds in the same way as it did with disjunction and *either* NPs: the *any* NP takes exceptional scope, as in (69), and the sentence is recursively exhaustified to yield the conjunctive inference, provided in (70), where we continue to

[14] Although *any* NPs and *either* NPs have similar distributions, these are not quite the same. In particular, *either* NPs, but not *any* NPs, are acceptable in universal modal sentences, as exemplified in (i). Accordingly, the two cannot be subject to the same licensing condition. Rather, *either* NPs could be classified as 'existential free choice items' (Chierchia 2013), and their distribution might be governed by exhaustification alone, while that of *any* NPs might be governed by silent *even* (cf. Crnič 2022). The behavior of *either* NPs must be studied further.

(i) a. Gali must have got into either university.

b. *Gali must have got into any university.

assume that only the existential subdomain alternatives are relevant for simplicity.

(69) [exh$_{C'}$ [exh$_C$ [any$_D$ IL school] [fewer than 1000 students got into ~~[any$_D$ IL school]~~]]]

(70) $[\![$[exh$_{C'}$ [exh$_C$ [any$_D$ IL school] [fewer than 1000 students got into ~~[any$_D$ IL school]~~]]]$]\!]$ =

$\bigwedge_{x \in [\![\text{Ivy Leaguge school}]\!]}$ card({x | student x got into x}) $<$ 1000

So far so good. But how does such an analysis square with the fact that the *any* NP is an NPI? And how does it square with the fact that *either* NPs have a similarly restricted distribution? Perhaps surprisingly, the analysis is not only compatible with these expressions being NPIs, it bolsters support for a specific approach to NPI licensing, one that takes NPI licensing to be environment-based.

NPI Licensing. NPIs like *any* NPs have a famously restricted distribution.[15] The classical statement of the condition on its distribution, introduced by Fauconnier 1975 and Ladusaw 1979, requires them to occur in the scope of downward-monotone operators. This condition is not met in the above examples since the exhaustification operator *exh* is not downward-monotone, but rather non-monotone, and there are no other potential licensers c-commanding the NPIs. The examples as analyzed above thus further challenge this so-called operator-based formulation of the NPI licensing condition.

Environment-based condition. A reformulation of the condition in terms of environments, however, straightforwardly captures the above facts about NPI indefinites in addition to their distribution elsewhere. The condition is provided in (71) (cf., e.g., Kadmon and Landman 1993). (As the licensing condition on *either* NPs seems to be more complex, see fn. 14, and perhaps cannot even be stated in an informative way, we focus on *any* NPs in the following.)

(71) An occurrence of *any* NP is acceptable iff it is dominated by a constituent that is (Strawson) downward-monotone with respect to its domain.

The condition is satisfied in all the examples discussed above in which NPIs take exceptional

[15] We assume that *either* NPs and *any* NPs are not ambiguous between indefinites (sometimes called their 'NPI occurrences') and universal quantifiers (sometimes called their 'free choice item' occurrences). See, e.g., Chierchia 2004, 2013, Dayal 2013, Crnič 2017, 2019, 2022 for extensive discussion and arguments for *any* NPs and, e.g., Kratzer and Shimoyama 2002, Menéndez-Benito 2010, Fălăuş 2014, Alonso-Ovalle and Menéndez-Benito 2020, Fălăuş and Nicolae 2022 for analyses of 'free choice items' as indefinites in several other languages.

scope and convey conjunctive meanings after exhaustification (see Crnič 2019, 2022 for an analogous discussion of more standard 'free choice occurrences' of *any* NPs).

Illustration. Consider again the LF of sentence (67) in (69) and its interpretation in (70). In the LF, the NPI *any Ivy League school* takes scope above a downward-monotone operator. Now, if the meaning computed in (70) holds of a situation, then replacing the domain of the NPI with any subdomain results in a meaning that also holds of that same situation, as stated in (72) (at least on the assumption that NPIs induce only subdomain and scalar alternatives, see, e.g., Krifka 1995, Chierchia 2013 for the proposal, and Crnič 2019, 2022 for a discussion of the necessity of this assumption). Namely, the variants of the structure (69) in which *any* ranges over a smaller subdomain we lose entailments pertaining to some of the elements in the initial domain of *any*, that is, we lose some of the conjuncts in (70). The condition in (71) is accordingly satisfied.

(72) For any non-empty D $\subseteq$ [[Ivy League school]],

$$(\bigwedge_{x \in [\![\text{Ivy Leaguge school}]\!]} \text{card}(\{\text{x} \mid \text{student x got into x}\}) < 1000)$$
$$\Rightarrow (\bigwedge_{x \in D} \text{card}(\{\text{x} \mid \text{student x got into x}\}) < 1000)$$

The analysis of NPIs in these examples resembles the now-deprecated universal quantifier analysis of NPIs (e.g., Quine 1960, Lasnik 1972), on which *any* NPs are analyzed as a kind of universal quantifier that must take scope above appropriate expressions. Importantly, though, in contrast to these earlier analyses, we are making no special assumptions about the nature of these expressions other than two uncontroversial ones: that they are indefinites and that they are subject to a condition like (71). Moreover, our treatment above constitutes only one of the possible parses of the sentences

(and a dispreferred one at that), and it cannot be applied in a host of other environments.[16,17]

4 Some predictions

The theory gives rise to dauntingly many predictions. We focus on some pertaining to the trapping of the scope of disjunction and indefinites: if their scope can be successfully limited to their surface positions, the conjunctive inferences that we discussed above should not be available anymore, at least not without the expressions also contributing their usual meanings in their surface positions.

[16]In relation to the analysis being applicable only in specific cases, consider for example the sentence in (i).

(i) Everyone who knows any girl is lucky.

If exceptional scope is assigned to the NPI in this sentence, it cannot be strengthened to convey a conjunctive inference (see Sect. 2.1 for disscusion). Accordingly, the parse in (iia) violates the condition in (71) – the contribution of the NPI matches that of other indefinites in upward-monotone environments, and so the sentence is not downward-monotone with respect to its domain on this parse. The sentence may hence only be parsed with the NPI in the restrictor of the universal quantifier, as provided in (iib).

(ii) a. #[any girl] [everyone who knows ~~[any girl]~~ is lucky]
b. [everyone who knows [any girl] is lucky]

[17]We have observed that NPI indefinites can escape islands in Sect. 2.2. If they take matrix scope, they are licensed as discussed in the main text. But they can also escape islands without taking matrix scope. This can be appreciated by looking at a variant of Reinhart's (1997) well-worn examples in (i) (see also, e.g., Farkas 1981, Abusch 1993):

(i) a. [Context: A math textbook contains 500 difficult problems. Every math grad student is required to pick a problem and study every analysis that solves it. Tali studied every analysis that solves the four-color theorem. Zali studied every analysis that solves the Poincaré conjecture. But, as always, Gali is an exception.]
b. Gali DIDN'T study every analysis that solves ANY problem mentioned in the book.

The sentence has the plausible meaning that there is no problem in the book such that Gali studied its every analysis. This meaning can be derived as in (ii). Note that no exhaustification is required in this configuration in order to satisfy the licensing condition on *any* NPs (see Crnič and Buccola 2019 for a related discussion).

(ii) a. [neg [[any problem] [Gali studied [every analysis that solves ~~[any problem]~~]]]]
b. $\neg\exists$x: problem x $\wedge$ $\forall$z: analysis that solves problem x $\rightarrow$ Gali studied z

4.1 Disjunctive *either*

Larson 1985 showed that the scope of disjunction can be trapped by an appropriately placed *either* that precedes and associates with disjunction. In setting the stage, he observed that the sentence in (73), where *either* is adjacent to the disjunction, allows for three scope disambiguations:

(73) Sherlock pretended to look for either a burglar or a thief.
Readings: ✓ pretend>look for>or, ✓ pretend>or>look for, ✓ or>pretend>look for

In contrast, the sentence in (74), in which *either* is separated from the disjunction by a verb and a preposition, allows only for one disambiguation, the disambiguation on which the disjunction takes scope at the level at which *either* occurs at surface form:

(74) Sherlock pretended to either look for a burglar or a thief.
Readings: ✗ pretend>look for>or, ✓ pretend>or>look for, ✗ or>pretend>look for

Larson's generalization about the scope trapping ability of *either* is, accordingly, that whenever *either* is not adjacent to the disjunction it associates with, the disjunction is assigned scope at the level at which *either* occurs at surface form. To the extent this generalization is correct, and cannot be obviated by exceptional scope construals of disjunction, we can employ it to trap disjunction in our examples. More specifically, to the extent disjunction in the antecedent clause of (75) lacks an exceptional scope construal (say, the antecedent clause cannot be followed by "I don't remember which"), it should not allow for an exceptional scope construal in (76) either, all else equal.

(75) If Gal either got into Cambridge or Oxford, he was lucky.

(76) Fewer than 1000 students have either got into Cambridge or Oxford.

If the scope of disjunction is indeed trapped in the scope of *most* in (76), the sentence should only have a low-scope disjunctive construal – it should be judged as false in our above scenario in which more than 1000 students got into Cambridge or Oxford, though each of the universities accepted fewer than 1000 students. This prediction is borne out. More specifically, to the extent speakers fail to access an exceptional scope construal of sentence (75), which indicates that the trapping with *either* is effective (this seems to generally be the case), they take sentence (76) to only convey that

jointly Cambridge and Oxford accepted fewer than 1000 students.[18,19]

4.2 Non-exceptional scope taking indefinites

There are indefinites that do not allow for exceptional scope construal. This holds, in particular, for modified numeral indefinites (see, e.g., Reinhart 1997, Winter 2002, Ch. 3, for discussion, and Liu 1990, Beghelli 1995, Corblin 1997 for related earlier observations). For example, the sentence in (77a) cannot convey the meaning that there is a plurality consisting of more than one student such that everyone who knows all of the students in that plurality is lucky (see Ruys 1992, Reinhart 1997, Winter 1997 on the low-scope of distributivity with plural exceptional scope indefinites). This means that the sentence in (77a) lacks the LF in (77b).

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[18] A similar state of affairs holds also for sentences with *most*, as exemplified in (i): the sentence is judged as false in our scenario in the main text, in which 3 boys play for team A and B, 2 boys play just for team A, and 2 boys play just for team B (that is, only 3 of the 7 boys play for both teams).

(i) Most kids who either play for team A or team B play for both teams.

The sentence can nonetheless give rise to a conjunctive inference (see Bar-Lev and Fox 2020, Sect. 8.3, for a derivation), which means that the difference with respect to example (25) in the main text is merely in that (i) entails a low-scope disjunctive interpretation. Thanks to a reviewer for discussion.

[19] A related prediction arises for the variants of the examples in the main text in which *either* would attach at the matrix level. We discuss the prediction in relation to an occurrence of disjunction in a downward-monotone environment, with a major caveat that the sentence is somewhat marked for independent reasons (cf. Larson 1985 on *either* and negation, Schwarz 1999 on missing quantificational subjects in non-adjacent *either* disjunctions). Consider the sentence in (i) – to the extent the sentence is acceptable, it does not entail a conjunctive inference.

(i) ?Either fewer than 1000 students got into Cambridge or into Oxford.

This is in line with our proposal. Namely, Schwarz 1999 argues that sentences in which *either* is separated from its associate disjunction involve ellipsis (see also Wu 2022). Accordingly, since (i) involves such a separation, the sentence should have a structure like (iia) (we skirt issues involving the interpretation of the second subject, cf. Schwarz 1999, Sect. 5). Importantly, the structure does not involve movement of disjunction, and should have a parallel conjunctive alternative, provided in (iib). This alternative should inhibit conjunctive strengthening, which would explain the absence of conjunctive strengthening. Thanks to a reviewer for discussion.

(ii) a. [either [fewer than 1000 students got into Cambridge] [or [~~fewer than 1000/pro got~~ into Oxford]]]

b. [fewer than 1000 students got into Cambridge] [and [~~fewer than 1000 students/pro got~~ into Oxford]]

(77) a. Everyone who knows more than one student is lucky.

b. #[more than 1 student] [everyone who knows ~~[more than 1 student]~~ is lucky]

Accordingly, we predict that *more than one* NPs cannot give rise to conjunctive inferences without contributing a low-scope existential meaning. For example, consider a scenario in which we have four 5-membered teams with three members in common and in which each of the two remaining kids on a team is on exactly two teams (thus, 4 of the 7 kids are on exactly 2 teams, 3 of the 7 are on all 4 teams). One configuration satisfying these conditions is in Figure 1.

kid 1: teams	A, B
kid 2: teams	A, B
kid 3: teams	A, B, C, D
kid 4: teams	A, B, C, D
kid 5: teams	A, B, C, D
kid 6: teams	C, D
kid 7: teams	C, D

Figure 1: One possible make-up of the different teams.

The sentence in (78) cannot be judged as a true description of this scenario. This means that while the sentence can be construed as conveying a conjunctive inference, it cannot do so without the indefinite contributing a low-scope existential meaning, as indicated in (79). This means that the conjunctive inference can only be derived in a more standard way (e.g., Bar-Lev and Fox 2020).

(78) Most kids who are on more than one team are on all teams.

(79) **Observed inferences of sentence (78):**

a. $\Rightarrow [\![\text{most}]\!](\{x \mid \text{kid x is on more than 1 team}\})(\{x \mid \text{kid x is on all teams}\})$

b. $\Rightarrow \forall X\text{: } [\![\text{two teams}]\!](X) \rightarrow [\![\text{most}]\!](\{x \mid \text{kid x is on X}\})(\{x \mid \text{kid x is on all teams}\})$

4.3 Scope, binding, and the analysis of indefinites

One standard method of trapping scope of an indefinite is by means of binding a pronoun that is dominated by the indefinite (see, e.g., Schwarz 2001, Brasoveanu and Farkas 2011, Demirok 2019, Charlow 2020). In particular, indefinites as well as other quantificational expressions that contain a

variable bound by another quantificational expression cannot take scope over that quantificational expression. In our cases, accordingly, if an indefinite contains a variable that is bound by a quantifier within an islands, its scope will be trapped within the island as well – no scoping out followed by conjunctive strengthening of the indefinite will be available. This is schematized in (80):

(80) #[an NP ... pro$_x$ ...] [... [$_{island}$ QP$_x$ ... ~~[an NP ... pro$_1$ ...]~~] ...]

All else equal, we may expect that in such configurations conjunctive inferences are necessarily accompanied by a low-scope existential contribution of indefinites.

Challenge. A reviewer invites us to consider the sentence in (81) on the bound reading of the embedded pronoun. Together with the editor, they observe that the sentence can be judged as true in a scenario that parallels our earlier ones: there are 7 kids in total, 3 kids admire both of their parents, 2 kids admire only their father, and the other 2 kids admire only their mother. In short, the sentence seems to exhibit our target entailment pattern, as stated in (82) – conjunctive strengthening with no existential contribution of the indefinite in its surface position.

(81) Most kids who$_i$ admire one of their$_i$ parents admire both.

(82) **(Un)observed inferences of sentences (25)-(81):**

a. $\not\Rightarrow$ ⟦most⟧({x | kid x admires x's mom or dad})({x | kid x admires x's mom and dad})

b. $\Rightarrow$ ⟦most⟧({x | kid x admires x's mom})({x | kid x admires x's mom and dad}) $\wedge$
⟦most⟧({x | kid x admires x's dad})({x | kid x admires x's mom and dad})

While the availability of this reading is *prima facie* at odds with the predictions of our proposal, this appearance arises primarily due to the simplified treatment of indefinites.

Dependent indefinites. Indefinites allow for a type of reading we have not discussed year – a reading on which the contribution of the indefinite varies with a c-commanding quantifier (see, e.g., Hintikka 1986, Kratzer 1998, Chierchia 2001, Winter 2002, 2004, Schlenker 2006). For example, the sentence in (83) can describe a situation in which Gal hopes that every student joins a team they want to be on, which can be brought out by a continuation like "namely, Tal the Cavs, Jo the Mavs, etc." The meaning does not correspond to either a surface scope nor to an exceptional scope interpretation of the sentence (see Winter 2002, Schlenker 2006, Endriss 2009 for discussion and motivation).

(83) Gal hopes that every student joins a team.

It is controversial what the precise distribution of such readings is and what their proper analysis is (see, e.g., Ebert 2020 for a recent review). One influential analysis takes such indefinites to denote Skolemized choice functions (see Sect. 2.3 above for choice functions and notational conventions). Skolemized choice functions are functions from individuals to choice functions: for each individual, a Skolemized choice function picks out for it an element from a set of individuals (e.g., Winter 2002, 2004, Schlenker 2006). Accordingly, sentence (83) may be analyzed as in (84), yielding the target reading of the sentence, on which teams vary with students.

(84) a. [$\exists_f$ [Gali hopes that every student$_x$ join a$_{f(x)}$ team]]

b. $\exists$f$\in$SCH: $\forall$x: student x $\rightarrow$ x joins f(x)({x | team x})

Turning back to the sentence in (81), an LF that yields such a dependent interpretation is provided in (85a), where we take the parameter of the Skolem choice function to be bound by the quantifier. The interpretation of the LF is given in (85b), where the choice of a parent depends on the kid.

(85) a. [$\exists_f$ [most$_x$ kids who admire one$_{f(x)}$ of their$_x$ parents admire both their$_x$ parents]]

b. $\exists$f$\in$SCH: [[most]]({x | kid x admires f(x)({x's mom, x's dad})
({x | kid x admires x's mom and dad})

If we assume that the Skolemized choice functions in the domain of existential closure are natural (cf. Sharvit 1999, Winter 2004, Schlenker 2006; but see, e.g., Endriss 2009 for qualifications), one can restate the meaning in (85) as in (86), where we take the pertinent choice functions to be from an individual to their mother and from an individual to their father. Importantly, on this construal, the sentence does not entail a low-scope interpretation of the indefinite.

(86) (85) $\Leftrightarrow$ [[most]]({x | kid x admires x's dad})({x | kid x admires x's mom and dad}) $\vee$
[[most]]({x | kid x admires x's mom})({x | kid x admires x's mom and dad})

The set of alternatives to the structure in (85a) is not closed under conjunction, and the structure may be strengthened to convey a conjunctive inference according to condition (C): namely, the conjunction of the two proper subdomain alternatives is not equivalent to the low-scope existential alternative, as witnessed by the scenario provided above. The is achieved by recursive exhaustifica-

tion, as indicated in (87), where the crucial role is again played by the two subomain alternatives (which correspond to the exhaustifications of the two disjuncts in (86)): if one of them is true, the other one is as well, and since one must be true, both of them are.

(87) $[\![[\text{exh}_{C'}[\text{exh}_{C}[\exists_f[\text{most}_x \text{ kids who admire one}_{f(x)} \text{ of their}_x \text{ par. admire both their}_x \text{ par.}]]]\!] =$

$[\![\text{most}]\!](\{x \mid \text{kid x admires x's dad}\})(\{x \mid \text{x admires x's mom and dad}\}) \wedge$

$[\![\text{most}]\!](\{x \mid \text{kid x admires x's mom}\})(\{x \mid \text{x admires x's mom and dad}\})$

We thus see that trapping by binding does not necessarily preclude generation of conjunctive inferences. We sketched an approach to these data that effectively treats respective indefinites as functional indefinites, and we modelled this with Skolemized choice functions. Now, if an analogous analysis is unavailable on the simple movement approach, something we cannot explore here, the data discussed here may provide us with grounds to adopt a more sophisticated, choice function treatment of indefinites. Fortunately, such a switch would leave much of our preceding discussion unaffected (see Sect. 2.3).[20]

5 Wrapping up with conditionals

Our starting-point was a prediction of the grammatical theory of exhaustification and alternatives: if a sentence that dominates disjunction or an indefinite lacks a parallel conjunctive or universal quantifier alternative, its set of alternatives might not be closed under conjunction, and the sentence might accordingly give rise to a conjunctive inference. We pursued this prediction by looking at sentences in which disjunction and indefinites took scope that is unavailable to their conjunctive and universal quantifier alternatives, specifically, sentences in which disjunction and indefinites took scope out of islands and over certain downward-monotone operators. The prediction was borne out.

The paper thus expanded the range of constructions in which disjunction and indefinites can undergo conjunctive strengthening, on the one hand, and made a further case for the explanatory potency of the grammatical theory of exhaustification and alternatives, on the other hand. But many questions and directions for further research remain. These include predictions pertaining to

[20]The reviewer's example in (81) is easily construed as having a dependent reading of the indefinite, and the functions in the domain of the existential closure are salient natural functions. It may well be that such construals always require salience of the pertinent natural functions in the context. Absent such salience, or natural functions altogether, we would then expect the conjunctive strengthening readings of the kind discussed in the main text not to be readily available.

strengthening in environments other than those discussed above. To slightly mitigate this, we discuss some predictions about the behavior of exceptional scope disjunction in the antecedents of conditionals in Sect. 5.1 (cf. Santorio 2018, 2020). The paper then concludes by discussing some issues involving the robustness and cancellability of conjunctive inferences in Sect. 5.2.

5.1 Conjunctive strengthening in conditionals

The interpretation of conditional sentences depends on the modal expression in the matrix clause of the conditional (e.g., Kratzer 1981, 1986). Although the antecedents of conditionals seem to constitute non-monotone environments in all of them (cf., e.g., Stalnaker 1968, Lewis 1973, Kratzer 2012, and von Fintel 2001 for a differing view), the conditionals differ with respect to what inferences they give rise to if disjunction or indefinites occur in their antecedents. We zoom in on the behavior of disjunction in three types of conditionals below, following Santorio 2018, 2020: *probably*, deliberative, and bare conditionals. We predict conjunctive inferences for exceptional scope disjunction in *probably* and deliberative conditionals, which have indeed been observed by Santorio 2018, 2020, and an exclusive inference for exceptional scope disjunction in bare conditionals, which has been observed by Charlow 2019. This state of affairs is conditioned by whether the conjunctive inferences entail the respective low-scope disjunctive alternatives (which are excludable in all three kinds of conditionals), rather than by non-closure under conjunction of the sets of alternatives (which fails to obtain with all three kinds of conditionals). The otherwise puzzling data involving disjunction and indefinites in conditionals thus follows naturally on the grammatical approach to exhaustification and does not motivate tinkering with the semantics of conditionals or disjunction.

(i) *Probably* conditionals

Santorio 2018 observes that a *probably* conditional sentence in (88) can convey the conjunctive inference without disjunction contributing a disjunctive meaning in its surface scope position, as stated in (89). More to the point, consider a situation in which Sarah buys 40 lottery tickets, namely, tickets from 31-70. When addressing the question whether Sarah won the lottery or lost it, we can judge sentence (88) to be true, even though its surface disjunctive interpretation would be false: namely, Sarah's 40 tickets constitutes the majority of tickets between 1-70, and between 31-100, but not between 1-100 (which is equivalent to between 1-70 or between 31-100; we assume that the total number of tickets is greater than 100).

(88) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

(89) **(Un)observed inferences of sentence (88):**

a. $\not\Rightarrow$ If the winning ticket is between 1 and 100, probably Sarah won

b. $\Rightarrow$ If the winning ticket is between 1 and 70, probably Sarah won $\wedge$
If the winning ticket is between 31 and 100, probably Sarah won

Before proceeding to the derivation, we should elaborate on the missing low-scope disjunctive meaning in (88), and on some complexities involving the interpretation of *probably* conditionals.

Missing disjunctive meaning and *probably*. A reviewer notes that weak construals of *probably* may be available in (88), that on such construals the low-scope disjunctive meaning of the sentence may be evaluated as true, and hence that our claim that disjunction takes exceptional scope may not be warranted (see Lassiter 2011 for a review of the context-dependency of expressions of uncertainty, including data that shows that *probably* need not always convey 'more than 50% likely'). They suggest several ways of controlling for the strength of the conditional in order to solidify the judgements about the target reading. We have already tried to implement one of their suggestions by fixing the salient question under discussion in our scenarios. More to the point, Lassiter (2011) notes that the strong reading of *probably* is facilitated in examples like ours if the sentence addresses the question of whether Sarah won or lost the lottery (and not, say, who won the lottery). Hence, we insisted on mentioning this question in our scenario. But we can be even more explicit, as in (90): given the question about Sarah winning, B's response can be evaluated as true in our scenario, while C's response cannot be.[21]

(90) A: Did Sarah win the lottery or did she not win the lottery?

B: If the winning ticket is between 1 and 70 or between 31 and 100,
probably Sarah won the lottery.

C: %If the winning ticket is between 1 and 100, probably Sarah won the lottery.

[21] While we are trying to show that the pertinent sentences with *probably* conditionals are indeed false on their low-scope disjunctive construals by fixing its value to 50%, it should be noted that an argument analogous to Santorio's can be constructed for whatever value other than 50% one may employ in the evaluation of *probably* conditionals, simply by appropriately revising the numbers in our scenario. For illustration, if the pertinent value in the evaluation of *probably* was merely 30%, the tickets bought by Sarah could range between 45 and 70, etc.

As some speakers may accommodate an implicit question under discussion even in explicit question-answer discourses like these, which could affect the interpretation of *probably*, the reviewer suggests another control – employing a less context-sensitive expression than *probably*. Two sentences containing such expressions are provided in (91), both of which can be judged as true in our scenario. This is not the case for the paraphrases corresponding to low-scope disjunctive construals of the sentences in (92).

(91) a. If the winning ticket is between 1 and 70 or between 31 and 100, more likely than not Sarah won.

b. If the winning ticket is between 1 and 70 or between 31 and 100, it is more than 50% likely that Sarah won.

(92) a. ^{F}If the winning ticket is between 1 and 100, more likely than not Sarah won.

b. ^{F}If the winning ticket is between 1 and 100, it is more than 50% likely that Sarah won.

In short, the sentences under discussion give rise to conjunctive inferences, while not entailing a low-scope disjunctive reading of the sentence. This parallels our observations about the behavior of disjunction in the preceding sections.[22] (For concreteness, we treat *probably* conditionals as expressing the proposition that the probability of the consequent holding conditional on the antecedent holding is greater than 50%, a value which may well vary by context, see, e.g., Yalcin 2010, Lassiter 2011 for a more sophisticated analysis, as well as fn. 21.)

[22]Another argument for the exceptional scope construal of disjunction in these cases could come from Hurford disjunction. In particular, Singh 2008 argues that disjuncts must be mutually inconsistent for disjunction to be felicitous. Turning to the sentence in (88), some speakers perceive the embedded disjunction to be infelicitous when it appears on its own, (i), while they perceive the sentence in (88) that dominates it to be felicitous. This could follow from disjunction taking matrix scope, in which case the two disjuncts could be made mutually inconsistent by means of exhaustification (conjunctive strengthening could nonetheless be achieved due to simpler disjunct alternatives featured in recursive exhaustification, disjunct alternatives that are mutually compatible), while this is not the case for disjunction when it appears on its own (see, e.g., Fox and Hackl 2006, Nouwen 2010, Mayr 2013 for reasons and discussion).

(i) %The winning ticket is between 1-70 or between 31-100.

However, as the reviewers pointed out, neither the judgments here, nor the facts involving Hurford disjunctions in embedded environments, are solid enough to constitute a convincing argument for an exceptional scope parse.

Scope and strengthening. The sentence in (88) can be parsed with disjunction taking exceptional scope and the sentence being recursively exhaustified, as in (93).

(93) a. If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

b. [exh$_{C'}$ [exh$_{C}$ [$_{S}$ [between 1-70 or between 31-100]
[probably [if W is ~~[between 1-70 or between 31-100]~~] [Sarah won]]]]]

Given our assumptions, the formal alternatives to the base sentence, S in (93b), consist of two disjunct alternatives, a low-scope disjunctive alternative, and a low-scope conjunctive alternative:

(94) ALT(S) = { [probably [if W is [between 1-70 or between 31-100]] [Sarah won]],
[probably [if W is between 1-70] [Sarah won]],
[probably [if W is between 31-100] [Sarah won]],
[probably [if W is [between 1-70 and between 31-100]] [Sarah won]] }

In line with our preceding discussion, the sentence lacks an exceptional scope conjunctive alternative, of the form provided in (95), as such alternatives are not generated by grammar. This means that the set of alternatives to the sentence is not closed under conjunction, as stated in (96): for example, the conjunction of the disjunct alternatives is not equivalent to the low-scope disjunctive alternative (as witnessed by our initial scenario in which the former is true but the latter is false), or any other alternative for that matter. Accordingly, conjunctive strengthening may be possible.

(95) [between 1-70 and between 31-100]
[probably [if W is ~~between 1-70 and between 31-100~~] [Sarah won]]
$\notin$ ALT(S)

(96) **Condition (C) admits potential conjunctive inference for (88):**
ALT(S) is not closed under conjunction.

The excludable alternatives in (94) are the low-scope disjunctive alternative and the low-scope conjunctive alternative; neither disjunct alternative is excludable. If we assume that no low-scope alternative is relevant, we obtain the meaning in (97) for the embedded exhaustification:[23]

[23]Unlike in the earlier examples, the assumption about pruning is not merely cosmetic here. In particular, if the alternatives are not pruned, conjunctive strengthening is inhibited – it cannot be that Sarah has more than half of the tickets between 1-70 and between 31-100, but not more than half of the tickets between 1-100 (low-scope disjunction)

(97) $[\![\text{exh}_C\ \text{S}]\!]$ = Pr(Sarah won | W is between 1-70) $>$ 0.5 $\vee$
Pr(Sarah won | W is between 31-100) $>$ 0.5

All the alternatives to the sister of the higher *exh* are excludable. If only the disjunct alternatives are relevant, matrix exhaustification yields the conjunctive inference, as provided in (98).[24]

(98) $[\![\text{exh}_{C'}\ [\text{exh}_C\ \text{S}]]\!]$ = Pr(Sarah won | W is between 1-70) $>$ 0.5 $\wedge$
Pr(Sarah won | W is between 31-100) $>$ 0.5

(ii) Deliberative conditionals

Santorio (2020) discusses sentence (99), an instance of a deliberative conditional, in relation to the following scenario (cf., e.g., Thomason 1981, Schroeder 2011, Cariani et al. 2013 on deliberative modal flavor): A six-sided die has been thrown and you are offered a bet on even or on odd. The bet pays $50 if you win, and costs you $75 if you lose. Gal tells you that the die landed on 2 or 4. Tal tells you that it landed on 3 or 5. You do not know whether either report is accurate. Given this scenario, Santorio reports that sentence (99) can be judged as true and providing solid advice.

(99) If Gal's report or Tal's report is accurate, you ought to place a bet.

Conjunctive and missing disjunctive meaning. On the reading under discussion, sentence (99) does not have a surface disjunctive interpretation, as stated in (100a). Namely, you should not place a bet if all that is known is that one of Gal and Tal is correct, that is, that the die landed on a number between 2 and 5 – all else equal, you are likely to lose more than you could gain (see Santorio 2020 for further discussion). On top of that, the sentence gives rise to the conjunctive inference in (100b).

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or between 31-70 (low-scope conjunction). The pruning is admitted by the extant conditions on pruning (e.g., it weakens the output of low exhaustification, cf. Crnič et al. 2015, Bar-Lev 2018).

[24] A different implementation of *exh*, say, that of Bar-Lev and Fox's (2020), succeeds in a similar fashion. In particular, as we noted in the main text, the low-scope disjunctive and conjunctive alternatives to S in (93b) are excludable. This means that the disjunct alternatives are not includable according to Bar-Lev and Fox's (2020) definition: it cannot be that both disjunct alternatives are true, while both excludable alternatives are false (as discussed in fn. 23). But a recursive application of *exh* (with the pruning discussed in fn. 23) can deliver the target reading, by the exclusion of the exhaustified disjunct alternatives. This means that exhaustification in the main text and its variant with inclusion proceed fully in parallel in the example at hand, with only exclusion playing an active role in the derivation. In all the earlier examples in the main text, a single application of Bar-Lev and Fox's *exh* derives conjunctive inferences. See, e.g., Bar-Lev and Fox 2020, Alxatib 2023, Degano et al. 2023 for a further comparison of the different formulations of *exh*.

(100) **(Un)observed inferences of sentence (99):**

a. $\not\Rightarrow$ If the die landed on a number between 2 and 5, you ought to place a bet.

b. $\Rightarrow$ If the die landed on 2 or 4, you ought to place a bet $\wedge$
If the die landed on 3 or 5, you ought to place a bet

Scope and strengthening. The sentence in (99) can be parsed so that disjunction takes exceptional scope and the sentence is recursively exhaustified, as in (101).

(101) [exh$_{C'}$ [exh$_C$ [GR or TR] [ought [[~~GR or TR~~] is accurate] [you place a bet]]]]

The conjunction of the disjunct alternatives is not equivalent to the low-scope disjunctive alternative, in particular, the conjunction of the disjunct alternatives does not entail the low-scope disjunctive alternative. This is supported by us accepting the sentence in (99) in the scenario described above, and rejecting its low-scope disjunctive variant, as summarized in (100a). Since the sentence lacks an exceptional scope conjunctive alternative, this means that conjunctive strengthening of the sentence may be possible according to condition (C).

(102) **Condition (C) admits potential conjunctive inference for (99):**
ALT([GR or TR] [ought [[~~GR or TR~~] is accurate] [you have place a bet]]])
is not closed under conjunction.

Unlike in the case of *probably* conditionals, however, it is controversial why precisely the low-scope disjunctive alternative does not follow from the conjunction of the disjunct alternatives (see, e.g., Kolodny and MacFarlane 2010 for related discussion). The theoretical details are not crucial for our purposes, though we sketch for concreteness a derivation that builds on the restrictor approach to conditionals. On this approach, what distinguishes deliberative *ought* conditionals from 'regular' *ought* conditionals is that the former are sensitive to an additional parameter, a contextually salient decision problem (that is, a set of actions available to the deliberating agent), which affects the ordering of the worlds (Cariani et al. 2013). This additional parameter effectively makes the worlds in the modal base in which a specific action in the decision problem is chosen tied with respect to their goodness, and this goodness is determined on the basis of worlds in the modal base in which the specific action yields the worst outcome. In our example, the decision problem consists of three actions: bet on even, bet on odd, and don't bet. With respect to the conjunctive inference, we get

that the optimal worlds in which Gal's report (2,4) is accurate are those in which you bet on even, and the optimal worlds in which Tal's report (3,5) is accurate are those in which you bet on odd. In all of these worlds, you indeed place a bet. In contrast, with respect to the low-scope disjunctive alternative, the optimal worlds in which Gal's or Tal's report (2-5) is accurate are those in which you do not place a bet (= you do not lose any money) since those are better than the worlds in which Tal's report (3,5) is accurate and you bet on even (= you lose \$75), and those in which Gal's report (2,4) is accurate and you bet on odd (= you lose \$75) – recall that this comparison is dictated by having to effectively look only at the worlds in the modal base in which a specific action compatible with the antecedent yields the worst outcome (see Cariani et al. 2013 for details and motivation). This means that the conjunction of the disjunct alternatives does not entail alternatives, as stated in (103) (where $\text{MIN}^{Dec}_{\leq}$ is meant to pick out the optimal worlds given the decision problem *Dec*). Accordingly, condition (C) admits strengthening on the proposed analysis of deliberative conditionals.

(103) $\big(\text{MIN}^{Dec}_{\leq}(\{\text{w} \mid \text{Gal's report is accurate in w}\}) \subseteq \{\text{w} \mid \text{you place a bet in w}\} \wedge$
$\text{MIN}^{Dec}_{\leq}(\{\text{w} \mid \text{Tal's report is accurate in w}\}) \subseteq \{\text{w} \mid \text{you place a bet in w}\}\big)$
$\not\Rightarrow \text{MIN}^{Dec}_{\leq}(\{\text{w} \mid \text{G or T's report is accurate in w}\}) \not\subseteq \{\text{w} \mid \text{you place a bet in w}\}$

Recursive exhaustification delivers this strengthening. At the first layer of exhaustification, we obtain the meaning in (104) if the excludable low-scope disjunctive alternative is not relevant (unlike in the case of *probably* conditionals, this assumption is merely cosmetic here, see fn. 23).

(104) $[\![[\text{exh}_C\ [\text{GR or TR}]\ [\text{ought}\ [\sout{\text{GR or TR}}\ \text{is accurate}]\ [\text{you place a bet}]]]\!] =$
$\big(\text{MIN}^{Dec}_{\leq}(\{\text{w} \mid \text{Gal's report is accurate in w}\}) \subseteq \{\text{w} \mid \text{you place a bet in w}\} \vee$
$\text{MIN}^{Dec}_{\leq}(\{\text{w} \mid \text{Tal's report is accurate in w}\}) \subseteq \{\text{w} \mid \text{you place a bet in w}\}\big)$

At the second layer of exhaustification, the exhaustified disjunct alternatives are excludable. This is witnessed by their joint exclusion being consistent, as indicated in (105) (with the exclusion of the exhaustified disjunct alternatives, we obtain the inference that if one of the disjunct alternatives is true, the other one is as well). This meaning corresponds to the target conjunctive inference.

(105) $[\![[\text{exh}_{C'}\ [\text{exh}_C\ [\text{GR or TR}]\ [\text{ought}\ [\sout{\text{GR or TR}}\ \text{report is accurate}]\ [\text{you place a bet}]]]\!] =$
$\text{MIN}^{Dec}_{\leq}(\{\text{w} \mid \text{Gal's report is accurate in w}\}) \subseteq \{\text{w} \mid \text{you place a bet in w}\} \wedge$
$\text{MIN}^{Dec}_{\leq}(\{\text{w} \mid \text{Tal's report is accurate in w}\}) \subseteq \{\text{w} \mid \text{you place a bet in w}\}\big)$

In conclusion, the observed conjunctive inferences of *probably* and deliberative conditionals were derived on the grammatical theory of exhaustification. Accordingly, the complex behavior exhibited by disjunction in conditionals does not require a revision of the analysis of conditionals or disjunction (*pace* Santorio 2018, 2020).

(iii) Bare conditionals

We conclude the discussion of conditionals by looking at the bare conditional sentences in (106). Charlow 2019 notes that if we assign exceptional scope to the disjunction in it, the sentence gives rise to the inference that exactly one of cake or salad is such that if Gali orders it, her parents are relieved. That is, the sentence gives rise to the negation of the conjunctive inference – which is the opposite of what we observed for *probably* and deliberative conditionals. (On the default, low-scope construal of disjunction, the sentence gives rise to a conjunctive inference, which is called 'Simplification of Disjunctive Antecedents', see Bar-Lev and Fox 2020, among many others.)

(106) If Gali ordered cake or salad, her parents are relieved.

The sentence can be assigned the structure in (107), in which disjunction takes exceptional scope and the sentence is exhaustified. The sister of *exh* has the set of alternatives provided in (108).

(107) [exh$_C$ [$_S$ [cake or salad] [if Gali ordered ~~[cake or salad]~~ her parents are relieved]]]

(108) ALT(S) = { [cake or salad] [if Gali ordered ~~[cake or salad]~~ her parents are relieved],
[if Gali ordered [cake or salad] her parents are relieved],
[if Gali ordered cake her parents are relieved],
[if Gali ordered salad her parents are relieved],
[if Gali ordered [cake and salad] her parents are relieved] }

In contrast to the universal quantification sentence whose inferences we discussed in Sect. 2.1, the set of alternatives in (108) is not closed under conjunction: in particular, the conjunction of the disjunct alternatives is not equivalent to any alternative in the set. Let us see why. We treat conditionals as conveying that the worlds closest to the actual one in which the antecedent holds the consequent holds as well (cf., e.g., Stalnaker 1968, Lewis 1973, Kratzer 1981). While the conjunction of the disjunct alternatives clearly entails the low-scope disjunctive alternative, the two are not equivalent, as stated in (109). Namely, imagine a setup in which only salad is such that if Gali or-

dered it, her parents are relieved, and in which Gali ordering salad holds in all the closest worlds in which Gali orders something. In this setup the conjunction of the disjunct alternatives is false, while the low-scope disjunctive alternative is true – hence, the latter does not entail the former.

(109) $\text{MIN}_{\leq}(\{\text{w} \mid \text{Gali ordered}_w \text{ cake}\}) \subseteq \big(\{\text{w} \mid \text{Gali's parents are relieved}_w\} \wedge$
$\text{MIN}_{\leq}(\{\text{w} \mid \text{Gali ordered}_w \text{ salad}\}) \subseteq \{\text{w} \mid \text{Gali's parents are relieved}_w\}\big)$
$\not\Rightarrow$ $\text{MIN}_{\leq}(\{\text{w} \mid \text{Gali ordered}_w \text{ cake or salad}\}) \subseteq \{\text{w} \mid \text{Gali's parents are relieved}_w\}$

Given that the conjunction of the disjunct alternatives is not equivalent to any other alternative either, we conclude that indeed (108) is not closed under conjunction.

(110) **Condition (C) admits potential conjunctive inference for (106):**
ALT([cake or salad] [if Gali ordered ~~[cake or salad]~~ her parents are relieved])
is not closed under conjunction.

But a potential conjunctive inference being in principle admitted does not mean that it can in fact be generated, that is, conditon (C) characterizes merely a necessary condition on the generation of conjunctive and universal inferences. Whether conjunctive strengthening can obtain hinges on exhaustification, in particular, on whether the conjunctive inference entails any excludable alternatives to the sentence. The sentence in (106) has two excludable alternatives: the low-scope conjunctive and the low-scope disjunctive alternative, as stated in (111).

(111) IE([cake or salad] [if Gali ordered ~~[cake or salad]~~ her parents are relieved]) =
$\big\{\text{MIN}_{\leq}(\{\text{w} \mid \text{Gali ordered}_w \text{ cake or salad}\}) \subseteq \{\text{w} \mid \text{Gali's parents are relieved}_w\},$
$\text{MIN}_{\leq}(\{\text{w} \mid \text{Gali ordered}_w \text{ cake and salad}\}) \subseteq \{\text{w} \mid \text{Gali's parents are relieved}_w\}\big\}$

The excludability of the low-scope disjunctive alternative is inhibitory with respect to conjunctive strengthening: the target conjunctive inference entails it, as stated in (109), so its exclusion entails the negation of the conjunctive inference, which consequently cannot be generated. Instead, the negation of the excludable disjunctive alternative entails the negation of the conjunctive inference, which corresponds to the target inference of sentence (106), namely, that not both cake and salad are such that if Gali ordered it, her parents are relieved:

(112) $[\![[\text{exh}_C\ [_S$ [cake or salad] [if Gali ordered ~~[cake or salad]~~ her parents are relieved]]]$]\!] \Rightarrow$

$$\neg\big(\textsc{min}_{\leq}(\{\text{w} \mid \text{Gali ordered}_w \text{ cake}\}) \subseteq \{\text{w} \mid \text{Gali's parents are relieved}_w\} \wedge$$
$$\textsc{min}_{\leq}(\{\text{w} \mid \text{Gali ordered}_w \text{ salad}\}) \subseteq \{\text{w} \mid \text{Gali's parents are relieved}_w\}\big)$$

The preceding discussion illuminated three aspects of the theory. First: while condition (C) provides a good approximation of the distribution of conjunctive inferences, and is a helpful guide when it rules out conjunctive strengthening, it is merely a necessary condition – whether conjunctive strengthening is indeed predicted for specific cases requires the computation of the exhaustified meaning. Second: conjunctive strengthening is tightly constrained by the alternatives to a sentence, an observation that we have already discussed in relation to universal quantification sentences. Third: the inferences accompanying exceptional scope elements, their scalar implicatures, can be derived without assuming that the exceptional scope disjunction or indefinite have exceptional scope conjunction or universal quantifier as alternatives. This is reassuring given our characterization of alternatives in (5) and given that these latter alternatives cannot be generated in grammar (see, esp., Demirok 2019, Charlow 2020 for two recent principled explanations of this).

Summary of the typology

We identified two types of conditionals in relation to what kinds of inferences disjunction that surfaces in their antecedents gives rise to. This typology is conditioned by the logical properties of the antecedents (cf. Santorio 2018, 2020). It has been previously established that if disjunction is interpreted as taking low-scope in conditionals, conjunctive strengthening obtains across the board, yielding so-called 'Simplification of Disjunctive Antecedents' inferences (see Bar-Lev and Fox 2020 for discussion and derivation). In contrast, if disjunction is interpreted as taking exceptional scope outside of the antecedent, only the *probably* and deliberative conditionals can give rise to conjunctive inferences, since bare conditionals induce alternatives that inhibit such strengthening.

(113) a. ***Probably* and deliberative conditionals with disjunctive antecedents:** exceptional scope construal of disjunction leads to conjunctive strengthening (and due to its exceptional scope there is no low-scope disjunctive contribution of disjunction)

b. **Bare conditionals with disjunctive antecedents:** exceptional scope construal of disjunction leads to an exclusive inference (and due to its exceptional scope there is no low-scope disjunctive contribution of disjunction)

5.2 Optionality vs. obligatoriness of conjunctive strengthening

In many cases discussed above, conjunctive strengthening is not obligatory. The non-conjunctive readings can be brought out by appending "I don't remember which" to the sentences, a continuation that is incompatible with conjunctive strengthening. For example, such a continuation is possible in (114), but not in its variant with *either* in (115).

(114) If the winning ticket is between 1-70 or between 31-100, probably Sarah won.
I don't remember which.

(115) #If the winning ticket is from either grouping, probably Sarah won. I don't remember which.

While the unacceptability of (115) is expected (conjunctive strengthening is necessitated in the first sentece by the licensing condition on *either*, see Sect. 3.3, and this strengthened meaning clashes with the continuation), the acceptability of (114) is challenging. While this may appear to be surprising – exhaustification need not always apply after all – the challenge becomes apparent if we assume, as one ought perhaps, that whenever conjunctive strengthening is possible, it obtains, which is stated in a brute force way in (116) (see Chemla and Singh 2014, Bar-Lev and Fox 2020, Gotzner and Romoli 2022 for an extensive review and refinements; see fn. 10 for a related discussion).

(116) **An approximation of a generalization:**
If an (unexhaustified) sentence has a parse that can be conjunctively strengthened, conjunctive strenghtening must apply to that parse.

To satisfy this generalization, sentences that have both a conjunctive reading and a non-conjunctive reading have been treated as having two substantially different parses. For example, consider the sequence in (117): the first sentence tends to give rise to a conjunctive inference when it appears on its own, but this can be suspened by an appropriate continuation, such as "I don't know which."

(117) You are allowed to have cake or soup. I don't know which.

The disappearance of the conjunctive inference in (117) has been attributed to the first sentence being parsed with wide-scope disjunction (see, e.g., Fusco 2019), as provided in (118) (the sentence may then even be strengthened to convey an exclusive meaning, assuming that the wide-scope conjunctive alternative is available here).

(118) [cake or soup] [you are allowed to have ~~[cake or soup]~~]

Thus, the acceptability of (117) is compatible with the generalization in (116): conjunctive strengthening simply cannot apply to the parse in (118), so it does not. This strategy is not obviously available to us in (114), however. Namely, we are already assigning the widest scope to disjunction. In this specific case, however, the target non-conjunctive reading can be derived simply by not pruning the low-scope disjunctive alternative in exhaustification (which is crucial for deriving conjunctive strenghtening, see fn. 23). But what about the cases in which somesuch pruning is of no avail, say, the first sentence in (119)?

(119) Most kids who are on team A or team B are on both teams. I don't know which.

Here is a possible direction. Let's assume that instead of the condition in (116), a slightly different condition governs strenghtening: each sentence must be recursively exhaustified, and pruning of disjunct alternatives is dispreferred (see Bar-Lev and Fox 2020 for a more sophisticated condition).

(120) **A different approximation of a generalization:**
Every sentence must be c-commanded by recursively applied exhaustification operators.

Now, what structures can we assign to the sentence in (119) in which disjunction would take exceptional scope? At least two come to mind. One we have already discussed extensively in this paper. Another one is provided in (121): it differs from the parses above merely in having a silent ASSERT prefix, which denotes a universal epistemic modal (it conveys that the speaker believes the sentence that is prefixed). Given the ever-growing list of reasons to assume such a prefix, the parse should be available (see, e.g., Alonso-Ovalle and Menéndez-Benito 2010, Chierchia 2013, Meyer 2013, Cohen and Krifka 2014, Krifka 2014, Beck 2016, among many others).

(121) [ASSERT [[teamA or teamB] [most kids who are on ~~[teamA or teamB]~~ are on both teams]]]

Recursive exhaustification of this structure, on the assumption that ASSERT does not have an existential modal alternative, does not yield a conjunctive inference, but rather the inference that the speaker is not certain which of the two teams is such that most kids on it are on both teams, as provided in (122) (cf. Bar-Lev and Fox 2020, Sect. 5.5, for related discussion, and Degano et al. 2023 for qualifications).

(122) $\neg\Box$(most kids on team A are on both teams) $\wedge$ $\neg\Box$(most kids on team B are on both teams)

The continuation in (119) is now compatible with the inferences of the first sentence. Whether this approach to the 'cancelling' of conjunctive strengthening in the examples at hand is viable requires further study, study that calls for another occasion.

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