Failure of Exhaustification with Inclusion

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Fox & Hackl (2006) discuss several cases of failure of exhaustification as well as its obviation by pruning of alternatives. This note draws some consequences of their proposal for the formulation of exhaustification that is enriched with inclusion (Bar-Lev & Fox, 2019).

1 Missing scalar implicatures and their reappearance

The sentence in (1) with a modified numeral quantifier *more than two children* fails to give rise to the scalar implicature that Mary does not have more than three children (e.g., Krifka 1999).

(1) Mary has more than two children.
$\not\rightsquigarrow$ $\neg$(Mary has more than three children)

In certain circumstances, however, scalar implicatures can be observed with modified numeral quantifiers. We review two types of such circumstances here.

Obviation by context. The first type of circumstance, exemplified below, involves an utterance of a sentence with a modified numeral quantifier in a context in which only some alternatives are relevant, perhaps due to an explicit question in the preceding discourse (Fox & Hackl 2006 construct such examples on Kroch's examples of obviation of negative islands). (Such obviation is sometimes particularly robust, as noted by Cummins et al. 2012; Enguehard 2018.)

(2) [We are discussing filing taxes. Having more than two children puts you in Bracket X. Having more than five children puts you in Bracket Y.]
Mary has more than two children.
$\rightsquigarrow$ $\neg$(Mary has more than five children)

Obviation by universal quantifiers. The second type of circumstance, also discussed in Fox & Hackl (2006), involves embedding a modified numeral quantifier under a universal quantifier. For example, the sentence in (3) can convey that not every parent here has more than three children, that is, that some parent here has exactly three children.

(3) Every parent here has more than two children.
$\rightsquigarrow$ $\neg$(Every parent here has more than three children)

The plot. Fox & Hackl (2006) provide a far-reaching account of these data. They show that on the assumption that all scales of measurement are dense, exhaustification will fail to yield scalar implicatures of modified numerals – unless certain alternatives are pruned, that is, taken to be irrelevant in the context. While pruning of alternatives is known to be crucial in examples like (2), which involve obviation due to contextual pressure, this has not been appreciated for examples of obviation by universal quantifiers like (3). This is due to the implicit, though unwarranted, assumption of fewer alternatives in such cases than are admitted on general theories of alternatives (e.g., Katzir 2007). We begin the note by rehearsing the derivations of the above data. We then turn to the question of how maintaining Fox & Hackl's proposal constrains the formulation of the exhaustification mechanism that is enriched with inclusion – that is, a formulation on which exhaustification can not only negate alternatives, but also assert them (Bar-Lev & Fox, 2019). Specifically, keeping certain parameters of the theory of exhaustification fixed, we argue that inclusion must be properly contradiction-free (while exclusion should not be). We show that one alternative proposal – a proposal that restricts inclusion to relevant alternatives instead of making inclusion properly contradiction-free – is not viable.

2 Failure of exhaustification and pruning

Exhaustification, alternatives, and pruning. Exhaustification is famuously restricted in what alternatives it may negate (see, e.g., Katzir 2007; Fox 2007; Singh 2008; Fox & Katzir 2011; Katzir 2014; Crnič et al. 2015; Trinh & Haida 2015; Breheny et al. 2018; Trinh 2019, among many others). A particularly influential approach to these issues has been put forward by Katzir (2007, 2014). He resolves the extant puzzles, in particular the symmetry problem and the problem of Hurford disjunctions, by making two assumptions. First, he restricts the pool of alternatives that the exhaustification mechanism has access to to those derived by substitution of constituents of the sentence with their sub-constituents and with selected other expressions:

(4) ALT(S) = {S′ | S′ is derived from S by substitution of its constituents with their sub-constituents or with lexical items}

Second, he restricts the pruning of alternatives – which corresponds to taking alternatives out of the computation – to those that would be excluded by exhaustification if no pruning applied. The alternatives that can be excluded are defined in (5) (Fox, 2007).

(5) $\text{Excl(S)} = \bigcap\{\text{M} \mid \text{M is a maximal subset of ALT(S) such that } \{\neg[\![\text{S'}]\!] \mid \text{S'}\in\text{M}\} \cup \{[\![\text{S}]\!]\} \text{ is consistent}\}$

The definition of the exhaustification device is in (6). (See Katzir 2014 for the advantages of this formulation of *exh*; cf. also Bar-Lev & Fox 2019; Crnič 2019b, and others.)

(6) $[\![\text{exh}_C\ \text{S}]\!] = [\![\text{S}]\!] \wedge \forall \text{S'} \in \text{Excl(S)} \cap \text{C}: \neg[\![\text{S'}]\!]$

Failure of exhaustification: unembedded cases. If we assume that all scales are dense, including what appear to be cardinal scales, we predict failure of exhaustification in (1)-(3) – unless certain alternatives are pruned. We illustrate this with an unembedded case of a modified numeral quantifier in (1) (cf. Gajewski 2009). Sentence (7) has the formal alternatives (8).

(7) [exh$_C$ [John has more than 2 children]]

(8) ALT([John has more than 2 children]) = {John has more than D children | D is a degree}

Assume that E is a maximal set of jointly negatable alternatives that is consistent with John having more than 2 children. E must be a proper subset of the set on the right of (9) since negating every alternative in that set would contradict John having more than 2 children (recall that we are assuming that even what appears to be a cardinal scale is dense). Both the set of formal alternatives in (8) as well as the set on the right of (9) are dense, as described in (10).

(9) $E \subset \{\text{John has more than D children} \mid D > 2\}$

(10) $\forall$p, r $\in$ ALT(John has more than 2 children): if p $\underset{\not\Leftarrow}{\Rightarrow}$ r,
then there exists an q $\in$ ALT(John has more than 2 children) such that p $\underset{\not\Leftarrow}{\Rightarrow}$ q $\underset{\not\Leftarrow}{\Rightarrow}$ r.

Now, assume that one alternative, *m*, is missing from E. If there are alternatives in E that are entailed by *m*, then E cannot be maximal since *m* can be negated consistently with the negation of all the alternatives in E. If *m* is not entailed by any alternative in E, and it asymmetrically entails the sentence, it can also be negated consistently with the negation of all the alternatives in E since the sentence can be true even if *m* is false (due to density). Thus, there cannot be a maximal set of jointly negatable alternatives in (8) that is consistent with John having more than 2 children. Given the definition of Innocent Exclusion in (5), this means that <u>every</u> alternative is Innocently Excludable, as given in (11). If all these alternatives are negated, we obtain a contradiction. This would account for missing scalar implicature in (1) (exhaustification would yield a contradiction, and could thus not apply). However, this would also make the scalar implicatures of (2)-(3) be unexpected. (See Gajewski 2009 for a more detailed derivation.)

(11) Excl(S) = $\bigcap\emptyset$ = the set of all alternatives

(12) $\bigcap A = \{S \mid \forall A'(A' \in A \rightarrow S \in A')\}$. Thus: if A = $\emptyset$, then $\bigcap$A = the set of all elements

Failure of exhaustification: embedded cases. The importance of pruning in the obviation by universal quantifiers is discussed in greater detail in Crnič (2019a). The crucial observation is that, on the one hand, every set of alternatives to (3) that can be jointly negated with the prejacent of *exh* being true can be expanded by another existential quantifier alternative of the form 'Some parent here has more than D kids,' for some degree D (which mirrors the property that was crucial for deriving exhaustification failure with unembedded cases of modified numeral quantifiers in the preceding paragraph). On the other hand, not all such alternatives can be jointly negated since that would contradict the meaning of the sister of *exh*. Consequently, since no maximal sets of jointly negatable alternatives exist, every alternative is Innocently Excludable, as was the case in (11). (Furthermore, in (3), having universal quantifier alternatives with numeral values smaller than 3, such as 'Every parent here has more than 2.2 children' will yield a contextual contradiction as well; such alternatives must then be pruned as well.)

Pruning. If certain alternatives are pruned in the above computations, consistent and observed readings are derived, as desired. In the cases of obviation by context, (2), this can be achieved by setting the resource domain of *exh* to (13-a). In the cases of obviation by universals, (3), this can be achieved by setting the resource domain *exh* to, say, (13-b).

(13) a. C = {John has more than 2 children, John has more than 5 children}
b. C = {Everyone has more than D children | D>2 and D natural number}

(14) **Pruning and obviation:** In order to obtain the observed obviations of failure of exhaustification, one must be able to prune the alternatives in exhaustification.

Let us now turn to a definition of exhaustification enriched with the notion of inclusion.

3 Inclusion

Inclusion. Bar-Lev & Fox (2019) revise the characterization of *exh* by adding a new clause to it, underlined in (15): in addition to negating relevant Innocently Excludable alternatives, which are defined as above, *exh* also asserts Innocently Includable ones. The Innocently Includable alternatives are defined in (16): they are the alternatives that are in all maximal sets of alternatives compatible with the negation of all the Innocently Excludable alternatives. The revision has several advantages over previous characterizations of *exh*, as Bar-Lev & Fox (2019) carefully show. (The definition in (15) differs from that of Bar-Lev & Fox in that it asserts prejacent separately from inclusion. This is crucial in order to capture the obviation facts described in the introduction, as we elaborate on below.)

(15) $[\![\text{exh}_C\ \text{S}]\!] = [\![\text{S}]\!] \wedge \forall \text{S'} \in \text{Excl(S)} \cap \text{C}: [\![\text{S'}]\!] = 0 \wedge \underline{\forall \text{S'} \in \text{Incl(S)}: [\![\text{S'}]\!] = 1}$

(16) $\text{Incl(S)} = \bigcap\{\text{M} \mid \text{M is a maximal subset of ALT(S)}$
$\text{such that } \{[\![S']\!] \mid S' \in \text{M}\} \cup \{\neg[\![S']\!] \mid S' \in \text{Excl(S)}\} \text{ is consistent}\}$

Failure of exhaustification revisited. The revised formulation of exhaustification also affects the treatment of exhaustification failure cases. In particular, it holds that if every alternative is Innocently Excludable, as noted in (11), then also every alternative is Innocently Includable. This holds because no sets of alternatives are compatible with the negation of all the alternatives (which corresponds to the set of Innocently Excludable alternatives in the cases at hand).

(17) Incl(John has more than 2 children) = $\bigcap \emptyset$ = the set of all alternatives

(18) **Fact:** If ALT(S) $\subset$ Excl(S), then ALT(S) $\subset$ Incl(S).

Consequently, it holds that the exhaustification in (1)-(3) will produce contradictory meanings on the revised characterization of *exh*, no matter what alternatives are taken to be relevant: even if the negation of the alternatives by *exh* fails to yield a contradiction (due to pruning), the assertion of all possible alternatives (say, [it's raining] and [it's not raining]) will yield one. This is undesirable given the acceptability and scalar implicatures of sentences (2)-(3).

4 Revising inclusion

Against pruning of includable alternatives. Unless one also revises the definition of inclusion, one cannot obviously restrict the set of Innocently Includable alternatives without inducing massive under- or overgeneration for the examples under discussion. First: Assume that the inclusion is restricted to the same alternatives as exclusion (provided by the resource domain of *exh*), as illustrated in (19)-(20). In this case, a contradiction would be unavoidable (unless no alternative was relevant, which is, again, undesirable given the scalar implicatures of (2)-(3)): since all alternatives are both Innocently Excludable and Innocently Includable, one would be asserting and negating the same alternatives.

(19) $[\![\text{exh}_C\ \text{S}]\!] = [\![\text{S}]\!] \wedge \forall \text{S'} \in \text{Excl(S)} \cap \text{C}: [\![\text{S'}]\!] = 0 \wedge \forall \text{S'} \in \text{Incl(S)} \cap \text{C}: [\![\text{S'}]\!] = 1$

(20) John has more than 2 children
$\wedge$ $\neg$John has more than 5 children (exclusion)
$\wedge$ John has more than 5 children (inclusion)

On the other hand, if includable alternatives were restricted by a different set of relevant alternatives, as in (21), one would run into massive overgeneration. For example, the sentence in (22) may be expected to convey that John has more than D children, for any D (assuming that no Innocently Excludable alternatives are relevant). We will not attempt to come up with an additional constraint that would adequately rein in this overgeneration of (21).

(21) $[\![\text{exh}_{C,R}\ \text{S}]\!] = [\![\text{S}]\!] \wedge \forall\text{S'} \in \text{Excl(S)} \cap \text{C}: [\![\text{S'}]\!] = 0 \wedge \forall\text{S'} \in \text{Incl(S)} \cap \text{R}: [\![\text{S'}]\!] = 1$

(22) John has more than 2 children. $\rightsquigarrow$ John has more than 10 children. (if relevant)

Furthermore, Bar-Lev & Fox (2019) provide further reasons against the contextual restriction of inclusion (e.g., free choice, which is due to inclusion, cannot be eliminated by context).

Properly contradiction-free inclusion. A more tenable revision of inclusion is to have it assert alternatives only if their joint assertion does not lead to a contradiction (which would be the case if all alternatives were Innocently Excludable). For the examples under discussion, this means that no alternatives would be included, which is desirable. (Note that this formulation of inclusion makes it necessary to assert the prejacent independently of the inclusion mechanism.)

(23) $\text{Incl(S)} = \bigcap\{\text{M} \mid \text{M is a maximal subset of ALT(S) such that } \{[\![S']\!] \mid S' \in \text{M}\} \cup \{\neg[\![S']\!] \mid S' \in \text{Excl(S)}\} \text{ is consistent}\}$ $\underline{\text{if Excl(S)} \subset \text{ALT(S)},}$

$= \emptyset$ $\underline{\text{otherwise}}$

A perhaps slightly less brute force characterization of inclusion is in (24): in addition to being in all maximal sets of negatable alternatives, an incudable alternative must be compatible with the negation of all excludable alternatives and the sister of *exh*. This is impossible if all alternatives are excludable (in this case, $\{\neg[\![S']\!] \mid S' \in \text{Excl(S)}\}$ is already inconsistent).

(24) $\text{Incl(S)} = \{S' \mid S' \in \bigcap\{\text{M} \mid \text{M is a maximal subset of ALT(S) such that } \{[\![S']\!] \mid S' \in \text{M}\} \cup \{\neg[\![S']\!] \mid S' \in \text{Excl(S)}\} \text{ is consistent}\} \wedge \{[\![S']\!]\} \cup \{\neg[\![S']\!] \mid S' \in \text{Excl(S)}\} \cup \{[\![\text{S}]\!]\} \text{ is consistent}\}$

Finally, the formulation in (24) cannot be transposed to exclusion since that would make it impossible to generate scalar implicatures for modified numeral quantifiers, even in obviation circumstances discussed in the introduction (namely, no alternatives would be excludable, rather than all alternatives as is the case on the current formulation of exclusion).

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