Scoping out and strengthening

Luka Crnič

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Disjunction sometimes, remarkably, gives rise to conjunctive inferences without apparently contributing a disjunctive meaning. We present new instances of this puzzling behavior, expanding on earlier observations of Santorio 2018 and Bar-Lev and Fox 2020. Rather than calling for a revision of the analyses of disjunction and of the constituents hosting them, we argue that this behavior manifests disjunction's two familiar characteristics: they can take exceptional scope, and they can be strengthened to convey conjunctive inferences. As the same patterns obtain with NPI indefinites, we conclude that these too can take exceptional scope, even of the widest kind.

1 Yet another puzzle about disjunction and indefinites

1. Santorio (2018) observes that the *probably* conditional sentence in (1) can convey the conjunctive meaning provided in (2a) without also conveying a (low-scope) disjunctive one, paraphrased in (2b). More to the point, consider a situation in which Sarah buys 40 tickets in a 100 ticket raffle, namely, the tickets from 31-70. We can judge sentence (1) as true in this situation, even though its surface disjunctive interpretation is false, which we indicate with a superscripted F in (2b): namely, Sarah's 40 tickets constitutes the majority of tickets between 1-70, and between 31-100, but not between 1-100 (which is equivalent to between 1-70 or between 31-100). (Throughout the paper, the accessibility of the target readings may be facilitated by putting focal stress on *or*.)

(1) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

(2) a. If the winning ticket is between 1 and 70, probably Sarah won $\wedge$

If the winning ticket is between 31 and 100, probably Sarah won

b. ^{F}If the winning ticket is between 1 and 100, probably Sarah won

That the conjunctive meaning in (2a) can indeed be obtained for sentence (1), as stated above, can be gleaned from the reasoning in (3): namely, even if we know that (2b) is false, we can still accept the conclusion in (3). This is possible only if the initial sentence in (3) gives rise to the conjunctive inference in (2a), while not giving rise to the inference in (2b).

(3) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won. The winning ticket is between 1 and 70. So probably Sarah won.

This is one instance of the puzzle at the heart of this note, which we dub 'Santorio's puzzle':

Santorio's puzzle: Certain sentences that have the surface form S[p or q] may convey a conjunctive meaning, coresponding to $[\![$S[p]$]\!]$ and $[\![$S[q]$]\!]$, without apparently conveying a surface disjunctive meaning, corresponding to $[\![$S[p or q]$]\!]$. Moreover, these sentences may be compatible with, and may even convey, the negation of this disjunctive meaning, corresponding to $\neg[\![$S[p or q]$]\!]$.

2. Santorio's puzzle is not restricted to *probably* conditionals, as was importantly observed by Bar-Lev and Fox (2020). They apply Santorio's schema to proportional quantifiers (see Bar-Lev and Fox 2020, fn.46). Consider the sentence in (4), which can correctly describe a situation in which there are two teams, team A and team B, each with 5 kids as members, and with 3 kids being on both teams (that is, 4 of the 7 kids are on a single team, and 3 of the 7 kids are on both teams). Given this setup, the sentence is true if only its conjunctive inference is taken into account, provided in (5a), as indeed each team is such that the majority of kids on it that are also on the other team (3 of the 5 kids). But the sentence is false if disjunction contributes its usual meaning to the setence, paraphrased in (5b), as only 3 of the 7 kids are on both teams.

(4) Most kids on team A or team B are on both teams.

(5) a. Most kids on team A are on both teams $\wedge$ Most kids on team B are on both teams

b. ^{F}Most of the 7 kids are on both teams.

3. Similar behavior of disjunction can be found in other non-monotone and in downward-monotone environments. Let's first look at disjunction in the scope of non-monotone *only*-DPs. Consider the scenario of the World Cup Qualifiers: there were several qualifying groups – one team advanced from smaller qualifying groups, while two teams advanced from bigger qualifying groups. Sentence (6) can describe this qualification setup for two smaller groups A and B. But it can do so if only its conjunctive inference is taken into account, provided in (7a), as indeed each group is such that only one team advanced from it. The sentence is false if disjunction contributes its usual meaning, paraphrased in (7b), as at least two teams advanced from any two qualifying groups.

(6) Only one team advanced from group A or group B.

(7) a. Only one team advanced from Group A $\wedge$ Only one team advances from Group B
    b. ^{F}Only one team advanced from the teams constituting the two groups A and B

A similar state of affairs obtains also with occurrences of disjunction in the scope of downward-monotone quantifiers like *fewer than four Nobel prizes*. For example, Croatia and Mexico each have 3 recipients of a Nobel prize, that is, they have 6 recipients jointly. This distribution of Nobel prizes can be described by the sentence in (8), which is true if only the sentence's conjunctive inference is computed, (9a), without disjunction contributing its usual meaning, paraphrased in (9b).

(8) Fewer than 4 Nobel prizes went to Croatia or Mexico.

(9) a. Fewer than 4 Nobel prizes went to Croatia $\wedge$ Fewer than 4 Nobel prizes went to Mexico
    b. ^{F}Fewer than 4 Nobel prizes went to the scientists coming from Croatia and Mexico

4. Finally, all the above examples with disjunction can be rephrased with indefinites instead of disjunction – and this includes NPI indefinites. For example, the sentences in (10)-(13), in which an *either* NP replaces disjunction, can convey meanings equivalent to those conveyed by sentences (1), (4), (6) and (8), respectively, and can be judged true in the same scenarios – that is, the sentences are able to convey universal inferences without apparently conveying their usual surface disjunctive meanings. (One can get the target readings in the following also if one replaces *either* with *one*.)

(10) If the winning ticket is from either of the two groupings, probably Sarah won.

Can convey: $\forall$X$\in${grp1-70, grp31-100}: if the winning ticket is from X, probably S won

(11) Most kids on either team are on both teams.

Can convey: ∀X∈{team A, team B}: most kids on X are on team A and team B

(12) Only one team advanced from either of the two groups.

Can convey: ∀X∈{group A, group B}: only one team advanced from X

(13) Fewer than 4 Nobel prizes went to either of the two countries.

Can convey: ∀X∈{Croatia, Mexico}: fewer than 4 Nobel prizes went to X

Moreover, the same readings can also be induced by *any*-NPs (these are marked in the preceding examples due to the competition with *either*, which presupposes that its domain contains two elements).[1] For example, consider Santorio's setup from above, but with three salient groupings of tickets – tickets between 1-70, between 15-85, and between 31-100. The setup can be correctly described by the sentence in (14), which contains an *any* NP instead of disjunction. (As with disjunction, the accessibility of the target readings may be facilitated by having focal stress on *any*.)

(14) If the winning ticket is from any of the three groupings of tickets, probably Sarah won.

Can convey: ∀X∈{grp1-70, grp15-85, grp31-100}: if W is from X, probably S won

Similarly, if we tweak Bar-Lev and Fox's setup by having three teams that have three members in common instead of just two, we can felicitously describe the situation with the sentence in (15).

(15) Most kids on any of the three teams are on all of them.

Finally, consider a shape sorter game for preschoolers, where each object is matched with a single opening, as determined by their geometric shapes. A feature of the game can be described by sentence (16), where again *any* should bear focal stress. On a low-scope existential construal,

[1] Although *any* NPs and *either* NPs have similar distributions, these are not quite the same. In particular, *either* NPs, but not *any* NPs, are acceptable in universal modal sentences, as exemplified in (i). Accordingly, the two cannot be subject to the same licensing condition. Rather, *either* NPs could perhaps be classified as 'existential free choice items' (Chierchia 2013), and their distribution might be governed by exhaustification, while that of *any* NPs might be governed by silent *even* (cf. Crnič 2022). It goes without saying that more thorough investigation of *either* NPs is necessary.

(i) a. Gali must buy a ticket from either of the two groupings.
b. *Gali must buy a ticket from any of the salient groupings.

the sentence in (16) is false (as every object fits into some opening). It is true if only its universal inference is computed, namely, that every opening is such that only one object fits into it.

(16) Only one object fits into any of the openings.

Can convey: $\forall$X$\in${square opening, circular opening, ...}: only one object fits into X

5. Now, how can disjunction and indefinites convey conjunctive and universal meanings without apparently conveying disjunctive and existential meanings? Our answer is conservative: they can't. We argue that disjunction and indefinites' failure to contribute disjunctive and existential meanings in the above examples is merely apparent. This appearance arises due to two independent properties of disjunction and indefinites: them being able to take exceptional scope (hence, they need not to contribute low-scope disjunctive and existential meanings), and them being able to be strengthened to, respectively, conjunctive and universal meaning in such exceptional scope configurations. This is worked out in Sect. 2. Sects. 3 and 4 explore some consequences of the proposal, providing further support for it. In Sect. 5, exceptional scope NPIs are discussed in greater detail. Sect. 6 concludes.

2 Scoping out and strengthening resolves the puzzle

2.1 Scoping out ...

1. Disjunction and indefinites allow for exceptional scope, that is, they can take scope out of islands (e.g., Fodor and Sag 1982, Rooth and Partee 1982, and many others). In the case of (1), the sentence can convey merely that one of the disjuncts is such that if it holds of the winning ticket, Sarah probably won. The reading can be brought out by the parenthetical "but I don't remember which." (Exceptional scope readings of disjunction may require focal stress on it, see Schlenker 2006.)

(17) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

Can convey: If the winning ticket is between 1 and 70, probably Sarah won $\vee$
If the winning ticket is between 31 and 100, probably Sarah won.

Similarly, if an indefinite replaces disjunction, as in (18), we can get a wide-scope existential reading that there is a grouping of tickets such that if the winning ticket is from that grouping, Sarah probably won. The reading can be brought out by the continuation "namely, from the second group, 31-100." Importantly for our purposes, on these construals, the sentences in (17) and (18) do not entail a low-scope disjunctive or existential reading – this makes up one half of Santorio's puzzle.

(18) If the winning ticket is from a grouping of tickets that was mentioned, probably Sarah won.

Can convey: ∃g∈{grp1-70, grp31-100}:

If the winning ticket is from group g, probably Sarah won

2. Conjunction and universal quantifiers, in contrast, do not allow for such exceptional scope (e.g., May 1977, 1985; but see Barker 2012 for apparent exceptions). For example, a wide-scope construal of the universal quantifier in (19), a slightly modified variant of (10), would yield a true description of the above scenario. However, the sentence is judged as false, namely, if the antecedent is true, Sarah certainly wins. This means that only a low-scope construal of the quantifier is possible.

(19) ^{F}If the winning ticket is from all the groupings, probably but not certainly Sarah won.

Cannot convey: ∀g∈{group 1-70, group 15-85 group 31-100}:

If the winning ticket is from group g, probably but not certainly Sarah won

3. We will assume that exceptional scope is achieved by movement of the disjunctive phrases and indefinites, movement that is not available to conjunctive phrases and other quantifiers.[2] Accordingly, the meanings of the sentences in (17)-(18) described above are derived from parses (20)-(21).

(20) [between 1-70 or between 31-100]$_P$ [probably [if W is P] [Sarah won]]

(21) [a group that was mentioned]$_x$ [probably [if W is from x] [Sarah won]]

More specifically, given that the domain of the indefinite in (21) corresponds to the disjuncts in (20), the meaning of (20)-(21) corresponds to (22): the probability of Sarah having won conditional

[2]Our choice of the theory of exceptional scope should make, we hope, the subsequent exposition more transparent. Any theory that (i) assigns exceptional scope to the existential quantification introduced by disjunction and indefinites, or something akin to it, and (ii) does not assign something similar to conjunction and universal quantifiers would do for our purposes. Hence, the choice function approaches with existential closure (e.g., Reinhart 1997, Winter 1997, Matthewson 1999) as well as more sophisticated movement approaches (e.g., Demirok 2019, Charlow 2020) would do. In contrast, Schwarzschild's (2002) theory could not be adopted as it assigns low scope to existential quantification, nor could Fodor and Sag's (1982) or Kratzer's (1998) theories as they eschew existential quantification altogether.

on the winning ticket being between 1-70 is greater than 50%, or the probability of Sarah having won conditional on the winning ticket being between 31-100 is greater than 50%.

(22) Pr(λw. S wins in w | λw. W is between 1-70 in w)) $>$ 0.5 $\vee$

Pr(λw. S wins in w | λw. W is between 31-100 in w)) $>$ 0.5

Importantly, this meaning does not entail the meaning that we obtain on a low-scope construal of disjunction, which is stated in (23): in the described scenario, in which (22) holds, the probability of Sarah having won conditional on the winning ticket being between 1-100 is below 50%.

(23) (22) $\not\Rightarrow$ Pr(λw. S wins in w | λw. W is between 1-70 or between 31-100 in w) $>$ 0.5

In summary: Disjunction and indefinites may be assigned exceptional scope, that is, scope outside adjunct and DP islands hosting them. In these cases they do not contribute, respectively, disjunctive and existential meanings within those islands. But the meanings of such structures are weak, all else equal – they do not entail the conjunctive and universal meanings from Sect. 1.

2.2 ... and strengthening resolves the puzzle

1. Disjunction and indefinites can give rise to, respectively, conjunctive and universal inferences in certain sentences. This is most famously the case in sentences like (24)-(25), which contain so-called free choice occurrences of disjunction and indefinites (see, e.g., Kamp 1973, Zimmermann 2000, Sæbø 2001, Kratzer and Shimoyama 2002, Simons 2005, Menéndez-Benito 2005, **?**, Aloni 2007, Fox 2007, Klinedinst 2007, among many others).

(24) The ticket may be from group A or group B.

Conveys: $\Diamond$(the ticket is from group A) $\wedge$ $\Diamond$(the ticket is from group B)

(25) The ticket may be from either/any group.

Conveys: $\forall$g: group g $\rightarrow$ $\Diamond$(the ticket is from group g)

2. We assume that the strengthening of sentences with disjunction and indefinites so that they entail conjunctive and universal meanings happens by means of recursive exhaustification.[3] Exhaustifica-

[3] As in our discussion of exceptional scope, our choice of theory that derives the inferences in (24)/(25) can be seen as a matter of convenience and may be to some extent inconsequential. In principle, any theory would do that does not

tion, defined in (26a), negates all relevant innocently excludable alternatives, which are derived from the formal alternatives to a sentence, ALT(S), as defined in (26b) (cf. Katzir 2014, Crnič et al. 2015).

(26) a. $[\![\text{exh}_C\ \text{S}]\!] = [\![\text{S}]\!] \wedge \forall \text{S}' \in \text{IE(S)} \cap \text{C}: [\![\text{S}']\!]=0$

b. $\text{IE(S)} = \bigcap \{\text{M} \mid \text{M is a maximal subset of ALT(S) st } \{\neg[\![\text{S}']\!] \mid \text{S}' \in \text{M}\} \cup \{[\![\text{S}]\!]\} \text{ consistent}\}$

Instead of rehearsing exhaustification on the well-worn examples in (24)-(25), let us turn straight to our examples in (17)-(18). We show that on the exceptional scope construal of these examples, exhaustification can yield conjunctive and universal inferences, respectively. The sentence in (17), repeated below, can be assigned the recursive exhaustification structure in (27).

(17) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

(27) [exh$_{C'}$ [$_{S'}$ exh$_C$ [$_S$ [between 1-70 or between 31-100]$_P$ [probably [if W is P] [Sarah won]]]]]

In order to determine the output of exhaustification, we first need to determine the formal alternatives to the structure. We assume that these consist of the two disjunct alternatives, the low-scope disjunction alternative, and the low-scope conjunctive alternative (in line with Katzir 2007):

(28) ALT(S) = { [probably [if W is between 1-70] [S won]],
[probably [if W is between 31-100] [S won]],
[probably [if W is between 1-70 or between 31-100] [Sarah won]]],
[probably [if [W is between 1-70 and between 31-100] [Sarah won]]]] }

Importantly, the sentence lacks an alternative in which conjunction takes exceptional scope, of the form provided in (29), as it is not generated by grammar (see the discussion in Sect. 2.1, §2).[4]

(29) #[between 1-70 and between 31-100]$_P$ [probably [if W is P] [Sarah won] $\notin$ ALT(S)

require the presence of modality to derive conjunctive inferences and that derives conjunctive inferences in all cases in which there are no inhibitory alternatives (that is, innocently excludable alternatives that the conjunctive meaning entails in the idiom of the main text). See Sect. 4.1 for further discussion as well as, e.g., Fox 2007, Chemla 2009, Franke 2011, Singh et al. 2016, Bar-Lev 2018 for a more in-depth treatment of these issues.

[4]The same would hold if we opted for a different theory of exceptional scope, potentially in a more principled way. For example, on the choice function approach, there is no universal closure over choice functions (but see Charlow 2019). On a more sophisticated movement approach, the exceptional scope conjunctive alternative would have a contradictory meaning or be equivalent to the low-scope conjunctive alternative (see Demirok 2019, Charlow 2020 for discussion).

The innocently excludable alternatives to S in (28) are the low-scope disjunctive alternative and the low-scope conjunctive alternative; no disjunct alternative is excludable. If we assume that the conjunctive alternative is not relevant at the level of embedded exhaustification, that is, not in C, we obtain the meaning in (31) for the embedded sentence S in (27).

(30) IE(S) = {[probably [if W is between 1-70 or between 31-100] [S won]],
[probably [if W is between 1-70 and between 31-100] [S won]]}

(31) $[\![\text{exh}_C\ \text{S}]\!] = \big(\text{Pr(Sarah won} \mid \text{W is between 1-70)} > 0.5 \vee$
$\text{Pr(Sarah won} \mid \text{W is between 1-70)} > 0.5\big) \wedge$
$\text{Pr(Sarah won} \mid \text{W is between 1-100)} \leq 0.5$

We now turn to the higher exhaustification in (27). The sister to the higher *exh* has the alternatives that are built on those that we assumed above, provided in (32): the two disjunct alternatives, the low-scope disjunction alternative, and the low-scope conjunctive alternative.

(32) ALT(S') = { [exh_C [probably [if W is between 1-70] [Sarah won]]],
[exh_C [probably [if W is between 31-100] [Sarah won]]],
[exh_C [probably [if W is between 1-70 or between 31-100] [Sarah won]]],
[exh_C [probably [if W is between 1-70 and between 31-100] [Sarah won]]] }

All of the alternatives in (32) are innocently excludable. If we assume that all of them are relevant, that is, in C', and that C lacks the conjunctive alternative, as we did above, the second level of exhaustification yields the conjunctive reading, as provided in (33): namely, the negation of a disjunct alternatives yields the inference that if a disjunct alternative is true, the other disjunct alternative or the low-scope disjunction alternative must also be true; and given that the low-scope disjunction alternative is false, as computed in (31), we get that the other disjunct alternative must be true. And since at least one of the disjuncts must be true, the conjunctive inference follows. Finally, the negation of the conjunctive alternative at the matrix level is vacuous: it merely conveys that either it is false (which cannot be the case), or that one of the disjunct alternatives is true (which is the case), or that the low-scope disjunctive alternative is true (which cannot be the case). Thus, exceptional scope construal of disjunction combined with recursive exhaustification derives Santorio's puzzle.[5]

[5] A different implementation of *exh*, say, that of Bar-Lev and Fox's (2020), succeeds in a similar fashion. In particular, as we noted in the main text, both the low-scope disjunctive alternative and the conjunctive alternative are innocently

(33) $[\![\text{exh}_{C'}\ \text{S}']\!]$ = Pr(Sarah won | W is between 1-70) $> 0.5 \wedge$

Pr(Sarah won | W is between 31-100) $> 0.5 \wedge$

Pr(Sarah won | W is between 1-100) ≤ 0.5

Before we turn to the various predictions and limits of the proposed account, let us practice the above derivation on two other environments and on indefinite examples introduced above (see also Crnič 2023 for an application of the analysis in the domain of donkey anaphora).

3. The sentence in (4), repeated below, may be assigned the structure in (34).

(4) Most kids on team A or team B are on both teams.

(34) [exh$_{C'}$ [exh$_C$ [$_S$ [team A or team B]$_x$ [most [kids on x] [are on both teams]]]]]

The low-scope disjunction alternative to S in (34) is innocently excludable (the low scope conjunctive alternative is tautologous). If all alternatives are relevant at both levels of exhaustification, we obtain the desired output that most kids on team A are on both teams as are most kids on team B. In addition, the sentence also entails that it is false that most of all the kids are on both teams, an inference that follows from the assumption that the disjunctive alternative is relevant.

(35) $|\{\text{x} \mid \text{kid x on team A and team B}\}| > \frac{1}{2} \times |\{\text{x} \mid \text{kid x on team A}\}| \wedge$

$|\{\text{x} \mid \text{kid x on team A and team B}\}| > \frac{1}{2} \times |\{\text{x} \mid \text{kid x on team B}\}| \wedge$

$|\{\text{x} \mid \text{kid x on team A and team B}\}| \leq \frac{1}{2} \times |\{\text{x} \mid \text{kid x on team A or team B}\}|$

4. Indefinites are analyzed similarly to disjunction, though instead of the disjunct alternatives we consider their equivalents, the so-called subdomain alternatives (alternatives in which the domain of the existential quantifier in the original sentence is replaced with one of its subdomains). For example, sentence (13), repeated below, may be assigned the structure in (36), where the *either*-phrase takes matrix scope and the sentence is recursively exhaustified.

excludable to S in (27). This means that the disjunct alternatives are not innocently includable according to Bar-Lev and Fox's (2020) definition: it cannot be that both disjunct alternatives are true, while both excludable alternatives are false (it cannot be that Sarah would have more than half of the tickets in group 1-70 and group 31-100, but neither in group 1-100 nor group 31-70). But a recursive application of *exh* can, however, deliver the target reading. As recursive application of *exh* is needed in either case, we opted for the simpler formulation of *exh* in the main text.

(13) Fewer than 4 Nobel prizes went to either of the two countries.

(36) $[\text{exh}_{C'}\ [\text{exh}_C\ [_S\ [\text{either of the two countries}]_x\ [\text{fewer than 4 NP went to x}]]]$

The sister to the lower *exh* has the innocently excludable low-scope disjunction alternative (the low-scope conjunctive alternative is entailed by the sister of *exh*); the subdomain alternatives are all innocently excludable at the second level of exhaustification, especially on the assumption that all alternatives are relevant at all levels of exhaustification. Their joint exclusion yields the conjunctive inference, that is, the exhaustification yields the desired output that fewer than 4 Nobel prizes went to Croatia and that fewer than 4 Nobel prizes went to Mexico.

(37) $|\{\text{x} \mid \text{Nobel prize x goes to Croatia}\}| < 4 \wedge$
$|\{\text{x} \mid \text{Nobel prize x goes to Mexico}\}| < 4 \wedge$
$|\{\text{x} \mid \text{Nobel prize x goes to Croatia or Mexico}\}| \geq 4$

In line with our above discussion, we have again set aside the wide-scope universal quantifier alternative, as stated in (38). This choice may not be as obvious as above, however, since the structure does not violate island constraints and should be accessible given our assumptions. In spite of that, the structure seems to be unavailable, as witnessed by *fewer than 4 Nobel prizes went to every country* not allowing for an inverse scope reading (similar considerations extend to the *only* examples; see Fleisher 2015, fn.25). This unavailability may well be a consequence of a condition on covert movement of universal quantifiers that requires it to weaken the meaning of the pertinent structure (see Mayr and Spector 2012, and Fleisher 2015 for an extensive discussion).

(38) $\#[[\text{every country}]_x\ [\text{fewer than 4 NP went to x}]] \notin \text{ALT(S)}$

This completes our account of the puzzle, which emerges from the interaction of two independent properties of disjunction and indefinites: their ability to take exceptional scope and their ability to be strengthened to conjunctive and universal meanings in certain configurations. In the following section, we provide another argument that the sentences under discussion indeed rely on the two properties. We then turn to some predictions of the account, in Sect. 4, and to exceptional scope NPIs, something that we breezed over in the preceding, in Sect. 5.

3 An argument from the Hurford Constraint

1. Not all pairs of predicates can be disjoined felicitously. For example, the sentence in (39) is grammatical but infelicitous. Its infelicity is standardly attributed to one disjunct entailing the other (a violation of the 'Hurford Constraint', see, e.g., Hurford 1974, Gazdar 1979, Chierchia et al. 2011).

(39) ?The winning ticket was bought in Paris or France.
Fact: W was bought in Paris $\Rightarrow$ W was bought in France

Singh (2008) argues for a stronger condition on disjunction: the disjuncts must be mutually inconsistent. An example supporting this stronger version of the condition is in (40), where the disjuncts are independent but not inconsistent (see Singh 2008 for a detailed discussion).

(40) ?The winning ticket was bought in Turkey or in Asia.
Fact: W was bought in Turkey $\not\Rightarrow/_{\not\Leftarrow}$ W was bought in Asia
Fact: W was bought in Turkey $\wedge$ W was bought in Asia $\neq \bot$

All examples with disjunction that we looked at in introducing Santorio's puzzle contain surface occurrences of disjunction that have a form similar to (40). In fact, if you consider unembedded occcurrences of some of these disjunctions, you get infelicitous sentences. This is exemplified for the case of (1), repeated below, in (41) – the sentence is as infelicitous as those in (39)-(40) (see Singh 2008, Sect. 4, for discussion of Hurford Constraint violations in embedded environments). The fact that the sentence in (1) is, in contrast, felicitous calls for an explanation.

(1) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

(41) ?The winning ticket is between 1 and 70 or between 31 and 100.
Fact: W is between 1 and 70 $\not\Rightarrow/_{\not\Leftarrow}$ W is between 31 and 100
Fact: W is between 1-70 $\wedge$ W is between 31-100 $\neq \bot$

2. The issue is fully general. For example, it can be replicated by incorporating the infelicitous sentence in (40) into a Santorio setup. Imagine again a raffle with 100 tickets, but this time it is a cross-continental affair: the tickets 31-70 were bought in Ankara, which is in Asia; the first 30 tickets were bought on the European side of Istanbul (not in Asia); and the final 30 tickets were bought in Beijing (not in Turkey). The sentence in (42) can provide a true, felicitous description of the situation, a description that is again true only if the sentence's conjunctive inference, but not the low-scope construal of disjunction, is taken into account (70 tickets were bought in Turkey; 70 tickets were bought in Asia; 100 tickets were bought in the combination of the two).

(42) If the winning ticket was bought in Turkey or in Asia, probably the winner is from Ankara.

The acceptability of (1) and (42) indicates that the disjunction cannot be interpreted in the antecedent of the conditional in these examples, as the sentences are felicitous. On the other hand, assigning exceptional scope to disjunction allows us to explain the acceptability of these sentences, while maintaining the Hurford Constraint in its strongest form and taking exceptional scope disjunction to be subject to it as well. (What we show below can be transferred to all other examples discussed above. One can read the preceding discussion as either assuming that the Hurford Constraint does not apply to exceptional scope disjunction, or as adopting its weaker formulation.)

3. Given the stronger formulation of the Hurford Constraint, some legwork must be done. Namely, on our analysis of disjunction in sentences like (1) and (42), repeated below, disjunction should violate Singh's formulation of the Hurford Constraint as much as it does in (41): namely, the two disjoined propositions are mutually consistent, as witnessed in (43) by their conjunction being consistent (that their conjunction is entailed by the sentence is, in fact, at the heart of the paper).

(27) [exh$_{C'}$ [$_{S'}$ exh$_C$ [$_S$ [between 1-70 or between 31-100]$_P$ [probably [if W is P] [Sarah won]]]]]

(43) Pr(Sarah won | W is between 1-70)>0.5 $\wedge$
Pr(Sarah won | W is between 31-100)>0.5 $\neq \perp$

4. In contrast to (40) and (41), however, another layer of exhaustification can rescue the disjunction in (1) and (42) (see Singh 2008, Chierchia et al. 2011 for discussion). While the exhaustification of the first disjunct in (41) is insufficient to bring about the required inconsistency with the second disjunct (see, e.g., Fox and Hackl 2006, Nouwen 2010, Mayr 2013 for reasons and discussion), the sentence in (1) can be parsed as in (44), with an embedded *exh* that can be assumed to range merely over the second disjunct alternative (that is, the embedded *exh* negates at most that if the winning ticket is between 31-100, probably Sarah won). In this, we are fully mimicking Singh et al.'s derivations of Hurford Constraint obviations. Given these assumptions, the meaning of S in (44) is provided in (45), where the two (wide-scope) disjuncts are indeed mutually inconsistent.

(44) [$\text{exh}_{C''}$ [$\text{exh}_{C'}$ [$_S$ [between 1-70 or between 31-100]$_P$ [exh_C [probably [if W is P] [S won]]]]]]

(45) $\big($Pr(λw. S won | λw. W is between 1-70)>0.5 $\wedge$
Pr(λw. S won | λw. W is between 31-100)$\leq$0.5$\big)$ $\vee$
Pr(λw. S won | λw. W is between 31-100)>0.5

The disjunct alternatives are now mutually inconsistent, satisfying the Hurford Constraint. But if they are mutually inconsistent, how can we derive the observed, consistent conjunctive inference?

5. As always, the output of matrix exhaustification in (44) depends on the alternatives over which *exh* ranges. The alternatives that we have been mechanically assuming so far for sentence (1) are updated in the first three rows of (46) (the update is the added embedded *exh*). However, in parallel to these updates, one also gets alternatives that are their simplifications (in line with Katzir 2007): namely, the disjunct alternatives in which the lowest *exh* is deleted, in the final two rows of (46).

(46) ALT(S) = { [probably [exh_C [if W is between 1-70] [S won]],
[probably [exh_C [if W is between 31-100] [S won]],
[probably [exh_C [if W is between 1-70 or between 31-100] [Sarah won]]],
[probably [exh_C [if W is between 1-70 and between 31-100] [Sarah won]]],
[probably [if W is between 1-70] [Sarah won]],
[probably [if W is between 31-100] [Sarah won]] }

All the alternatives that are exhaustifications of these (that is, all the alternatives to the sister of the highest *exh*) are innocently excludable at the level of the highest *exh*. This hold, in particular, for the exhaustifications of the last two alternatives in (46), whose negation then yields the conjunctive inference, as in the preceding section. Accordingly, Santorio's puzzle is resolved in a way that straightforwardly explains some otherwise puzzling obviations of the Hurford Constraint.

4 Inhibition and trapping

Most occurrences of disjunction and indefinites do not fit Santorio's schema. In some cases, the strenghtening is blocked because the sentence's alternatives gum it up; in other cases, Santorio's puzzle fails to emerge due to the unavailability of exceptional scope construals of indefinites. We outline some of these limits on the interaction of exceptional scope and strengthening below.

4.1 Inhibitory alternatives

1. Disjunction and indefinites do not always give rise to conjunctive and universal meanings. In particular, if an innocently excludable alternative has the targeted conjunctive or universal meaning, or is entailed by it, no strengthening to conjunctive or universal meaning is possible; rather, their negation is generated (e.g., Fox 2007, Chemla 2009, Franke 2011, Crnič et al. 2015, Singh et al. 2016, and others). For example, consider the universal modal sentence in (47). It cannot convey the conjunctive meaning that both group A and B are such that Gali is required to get a ticket from them:

(47) Gali is required to get a ticket from group A or group B.

Cannot convey: $\Box$(G gets a ticket from group A) $\wedge$ $\Box$(G gets a ticket from group B)

The absence of the conjunctive reading follows from the characterization of *exh* that we adopted. The sentence in (47) can be parsed either as in (48a), with disjunction taking low scope, or as in (48b), with disjunction taking high scope (the highest *exh* may be vacuous, hence the parentheses).

(48) ([exh$_{C'}$) [exh$_C$ [$_S$ required [Gali gets a ticket from group A or group B]]]

(49) ([exh$_{C'}$) [exh$_C$ [$_S$ [group A or group B]$_x$ [required [Gali gets a ticket from x]]]]

Neither parse of the sentence yields a conjunctive meaning: namely, exhaustification ranges therein over an innocently excludable alternative with an embedded conjunction, as provided in (50). This conjunctive alternative blocks subsequent strengthening to a conjunctive meaning. To better understand our subsequent derivations, it is instructive to see how.

(50) IE(S) = {[required [Gali gets a ticket from group A and group B]], ...}

If the alternative is relevant at the level of the lower *exh*, we get the inference that the conjunctive alternative is false, hence the target conjunctive meaning cannot be derived (note that conjunction and universal modals are commutative). If the alternative is not relevant, it nonetheless prevents the disjunct alternatives from being innocently excludable at the level of the higher *exh*: their joint negation, but not the negation of just one of them, contradicts the base meaning of the sentence and the negation of the conjunctive alternative, so they cannot be in all maximal sets of excludable alternatives. Accordingly, exhaustification in (48)-(49) does not yield the conjunctive inference.

2. Let us now turn to examples more closely related to the ones discussed in this paper. In his pioneering work on the scalar implicatures of exceptive scope elements, Charlow (2019) discusses various inferences of bare conditional sentences like (51). He observes that on the exceptional scope construal of the indefinite, the sentence tends to convey one of two types of inferences, described in the (a)- and (b)-lines of (51): the weaker inference is that it is false that every ticket is such that if Gali gave it to Tali, Tali won one prize, provided in (51a); the stronger inference is that there is exactly one ticket such that if Gali gave it to Tali, Tali won one prize, provided in (51b). Relevantly, both inferences contradict the universal inference of the kind that we derived in the preceding section, with *probably* conditional sentences. This turns out to be expected, however.

(51) If Gali gave Tali a ticket, Tali won one of the prizes.

a. $\rightsquigarrow_1$ $\neg\forall$x: ticket x $\rightarrow$ if Gali gave Tali ticket x, Tali won one prize

b. $\rightsquigarrow_2$ $\exists!$x: ticket x $\wedge$ if Gali gave Tali ticket x, Tali won one prize

3. The sentence in (51) can be assigned the structure in (52), in which the indefinite takes exceptional scope and the sentence is exhaustified. The alternatives that the lower *exh* ranges over are provided in (53): the matrix existential quantifier subdomain alternatives, the low-scope existential quantifier subdomain alternatives, and the low-scope universal quantifier subdomain alternatives.

(52) ([exh'_C]) [exh_C [$_S$ [a_D ticket]$_x$ [if Gali gave Tali x Tali won one prize]]]

(53) ALT(S) = { [$\text{a}_{D'}$ ticket]$_x$ [if Gali gave Tali x Tali won one prize]]],
[if Gali gave Tali $\text{a}_{D'}$ ticket Tali won one prize],
[if Gali gave Tali $\text{every}_{D'}$ ticket Tali won one prize] | $[\![\text{D'}]\!] \subseteq [\![\text{D}]\!]$ }

Again, a sentence with an exceptionally construed indefinite lacks the alternative with a universal quantifier occurring in a position corresponding to that of the indefinite. In contrast to our discussion in the preceding section, however, the absence of a wide-scope universal quantifier alternative plays no role in the example at hand, that is, it does not open the door to deriving the universal inference. We show this in the process of deriving the two inferences in (51).

4. In the embedded sentence S in (52), the low-scope existential quantifier alternatives, provided in the second row of (53), are all innocently excludable if their domains contain at least two tickets:

(54) IE(S) = {[if G gave Tali $a_{D'}$ ticket Tali won one prize] ... | ... card([[D']]∩[[ticket]])≥2 ...}

Unlike in the preceding section, however, the innocent excludability of these alternatives is inhibitory with respect to universal strengthening: their joint exclusion entails that just one ticket is such that if Gali gave it to Tali Tali won one prize. If all these alternatives are relevant, the sentence entails their exclusion, and this blocks universal strengthening. In this case, the meaning of (52) matches the second, stronger inference described in (51b). It is spelled out in (55), where we assume a variably strict analysis of bare conditionals, on which a conditional is true if in the closest world(s) in which the antecedent is true the consequent is true as well (all low-scope non-singleton universal quantifier alternatives may be negated as well, but we assume for brevity that they are not relevant). The strong inference in (51b) is entailed by the meaning in (55) because we get that for every pair of tickets, one of the closest worlds in which Gali gave Tali one of them is such that Tali did not win in that world; and since this holds, in particular, for every pair containing the ticket verifying the matrix quantification, we get that every other ticket is such that it is false that if Gali gave it to Tali, Tali won one prize (see Charlow 2019 for a discussion that makes different assumptions).

(55) $\big(\exists$x: ticket x in D $\wedge$ $\textsc{Min}_{\leq}(\lambda$w. G gave T x in w$)$ $\subseteq$ λw. T won one prize in w$\big)$ $\wedge$
$\forall$D$'\subseteq$D $\cap$ ticket: card(D$'$)$\geq$2 $\rightarrow$
$\neg\big(\textsc{Min}_{\leq}(\lambda$w. $\exists$x: ticket x in D$'$ $\wedge$ G gave T x in w$)$ $\subseteq$ λw. T won one prize in w$\big)$

On the other hand, if only the low-scope existential quantifier alternative with the same domain as in the sentence (that is, D) is relevant, we obtain a weaker meaning, namely, that not every ticket is such that if Gali gave it to Tali, Tali won one prize. This follows from the fact that the negation of the low-scope existential quantifier alternative simply means that there must be a ticket distinct from the one that verifies the matrix quantification and a closest world in which Gali gives Tali that ticket and Tali didn't win a prize. Accordingly, the inference described in (51a) is derived.

(56) $\big(\exists$x: ticket x in D $\wedge$ $\textsc{Min}_{\leq}(\lambda$w. G gave T x in w$)$ $\subseteq$ λw. T won one prize in w$\big)$ $\wedge$
$\neg\big(\textsc{Min}_{\leq}(\lambda$w. $\exists$x: ticket x in D $\wedge$ G gave T x in w$)$ $\subseteq$ λw. T won one prize in w$\big)$

Finally, if none of the low-scope existential quantifier alternatives with two tickets in the domain of the quantifier are relevant, we compute no additional inferences, as they prevent the subdomain alternatives to be innocently excludable, that is, to be in all maximal sets of excludable alternatives, along the lines discussed in §1 above: it cannot be that two singleton subdomain matrix existential quantification alternatives are true, while the innocently excludable low-scope existential closure alternative with the domain that is a union of these two singleton domains is false.

5. We have seen two things. On the one hand, the conjunctive and universal strengthening is tightly constrained by what alternatives the sentences under consideration have, an observation that has already received extensive attention in the literature. We showed one case of inhibition of universal strengthening of exceptional scope indefinites by switching from *probably* to bare conditionals. On the other hand, we have seen that the inferences accompanying exceptional scope elements, their scalar implicatures, can in many cases be correctly derived even without assuming that the exceptional scope disjunction or indefinite have exceptional scope conjunction or universal quantifier as alternatives. This is reassuring since these tend not to be generatable on most approaches to exceptional scope (e.g., Demirok 2019, Charlow 2020 for two recent examples).

4.2 More on inhibitory alternatives and relevance

1. Sentences exemplifying Santorio's puzzle allow for readings other than the one we have been discussing. For example, when introducing exceptional scope (Sect. 2.1, §1), we noted that (1) can convey the wide-scope disjunctive meaning, namely, that one of the conditional disjuncts is true. Moreover, the sentence can in addition convey the strengthened meanings discussed in the preceding subsection, following Charlow (2019), namely, that exactly one of the conditional disjuncts is true.

(17) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

Can convey: If the winning ticket is between 1 and 70, probably Sarah won $\boxed{\vee}$
If the winning ticket is between 31 and 100, probably Sarah won.

How can this meaning be derived given our above assumptions?

2. We can derive the observed meaning from the structure in (27), repeated below, by merely assuming that the low-scope conjunctive alternative is relevant at the embedded level. (We are again ignoring the strong Hurford Constraint in the following in order to have simpler representations.)

(27) [exh$_{C'}$ [$_{S'}$ exh$_C$ [$_S$ [between 1-70 or between 31-100]$_P$ [probably [if W is P] [Sarah won]]]]]

We have seen that the low-scope conjunctive alternative is innocently excludable. Assuming that all the alternatives are relevant, including it, this means that the meaning of S' in (27) is as follows:

(57) $\big(\text{Pr(S won} \mid \text{W is between 1-70)} > 0.5 \vee \text{Pr(S won} \mid \text{W is between 31-100)} > 0.5\big) \wedge$
$\text{Pr(S won} \mid \text{W is between 1-70 or between 31-100)} \leq 0.5 \wedge$
$\text{Pr(S won} \mid \text{W is between 1-70 and between 31-100)} \leq 0.5$

Consider now the meanings of the disjunct alternatives in the domain of the higher *exh*, one of whose meanings is provided in (58). These alternatives are not innocently excludable, that is, neither of them is in all maximal sets of excludable alternatives. For example, the negation of (58) together with the meaning in (57) entails that the other disjunct alternative is true.

(58) $\text{Pr(S won} \mid \text{W is between 1-70)} > 0.5 \wedge$
$\text{Pr(S won} \mid \text{W is between 31-100)} \leq 0.5 \wedge$
$\text{Pr(S won} \mid \text{W is between 1-70 or between 31-100)} \leq 0.5 \wedge$
$\text{Pr(S won} \mid \text{W is between 1-70 and between 31-100)} \leq 0.5$

This means that we are bound to the meaning provided in (57), which is true only if just one of the disjunct alternatives is true. We thus derived the desired interpretation, one that contradicts the conjunctive interpretation, merely by modifying our assumptions about relevance.

3. It is instructive to reflect on the conspicuous difference between the conjunctive alternative to disjunction in universal modal sentences, which blocks conjunctive strengthening no matter whether it is relevant or not (Sect. 4.1, §1), and the conjunctive alternative to disjunction in *probably* conditionals, which blocks conjunctive strengthening only if it is relevant at the level of lower exhaustification, discussed in this section. In both cases, all maximal sets of excludable alternatives contain the conjunctive alternative. In the former, universal modal case, the conjunctive alternative is the same at both levels of exhaustification – it is a simple universal modal sentence in which disjunction has been replaced by conjunction. It hence blocks conjunctive strengthening by preventing exclusion of strengthened disjunct alternatives at both levels. In the latter, *probably* conditional case, the conjunctive alternative at the lower level contradicts conjunctive strengthening, but can be assumed to be irrelevant; the conjunctive alternative at the higher level of exhaustification is, however, stronger than at the lower level: it is conjoined with the negations of the disjunct and disjunction alternatives (due to the sentence being non-monotone with respect to its antecedent). And its negation at that level is compatible with the conjunctive strengthening – in fact, it is entailed by it.

4.3 Scope trapping

1. Santorio's puzzle was shown to follow from an interaction of two independently motivated mechanisms: the exceptional scope assignment to disjunction and indefinites, and the strengthening of the resulting structures. In preceding subsections, we looked at representative cases in which strengthening did not yield a conjunctive/universal reading due to inhibitory alternatives. In the following, we look at predictions pertaining to trapping the scope of indefinites. The method we rely on is introducing a pronoun that is bound in the embedded clause into the indefinite phrase (see, e.g., Schwarz 2001, Brasoveanu and Farkas 2011, Demirok 2019, Charlow 2020).

2. Consider a setup along the lines of Santorio's in Sect. 1, but extended to three of my friends: that is, my friends participated in different raffles, each with a different number of total tickets; each friend's tickets could be split between two groupings jointly consisting of the total set of tickets in the raffle, so that the tickets they bought would be the majority of the tickets in each grouping, but not in the total set of tickets; and, finally, they discussed their groupings with me, to make sure they didn't make a mistake. Given this, the sentence in (59) is judged as false.

(59) ^{F}If everyone of my three friends$_i$ saw that a ticket from the groupings they$_i$ discussed with me was drawn, probably at least two of them won.

This is predicted on our proposal, as the indefinite containing the bound pronoun cannot scope above the binder in (59), and accordingly the sentence entails that if everyone of my three friends saw a ticket be drawn, at least two of them probably won. This inference is false in the described scenario – the probability of at least two of my friends winning is less than 40%. On the other hand, if the universal inference could be computed for (59) without deriving the low-scope existential interpretation of the indefinite, the sentence should be true in the described scenario.

2. This type of pattern can be replicated with other types of examples we discussed. For example, recall that the sentence in (12), repeated below, can convey that only one team advanced from group A and only one team advanced group B (the domain of *either* is constituted by these two groups).

(12) Only one team advanced from either of the two groups.

Now, consider a variant of the example: we have cotemporaneous qualifications for the World Cup and the European Championships. If I wanted to convey that only one team qualified for both events, that is, advanced from both qualifying groups it was in, I cannot do it by employing the sentence in (60), which can only convey the odd meaning that only one team qualified to one of the events. In short, universal strengthening cannot be accomplished in case the indefinite contains elements that are bound in the local clause in which the indefinite surfaces. This follows on our approach from the fact that such indefinites cannot be assigned exceptional scope.

(60) ^{F}Only one team$_i$ advanced from either of its$_i$ groups.

5 Exceptional scope NPIs more generally

Exceptional scope NPIs may appear to violate the common assumptions about NPIs. By discussing the limits and repercussions of their exceptional scope, we show that this appearance is misleading.

5.1 Exceptional matrix scope

*Any NP*s have a famously restricted distribution. It is captured well enough for our purposes by the licensing condition in (61) (cf. Kadmon and Landman 1993). (As the licensing condition on *either* NPs seem to be more complex, see fn.1, we focus on *any* NPs in the following.)

(61) An occurrence of *any NP* is acceptable iff it is dominated by a constituent that is (Strawson) downward-monotone with respect to its domain.

The condition is satisified in all the examples discussed above in which NPIs take exceptional scope and convey universal meanings after exhaustification. Consider, for example, the sentence in (14), repeated below, which is assigned the LF in (62) – in which the NPI *any of the three groupings of tickets* takes exceptional scope outside an adjunct island – and its interpretation in (63).

(14) If the winning ticket is from any of the three groupings of tickets, probably Sarah won.

(62) [exh$_{C'}$ [exh$_C$ [any of the three groupings]$_x$ [probably [if W is from x] [S won]]]]

(63) Pr(λw. Sarah won | λw. W is from group 1-70) $>$ 0.5 $\wedge$

$$\Pr(\lambda w.\ \text{Sarah won} \mid \lambda w.\ \text{W is from group 15-85}) > 0.5 \wedge$$
$$\Pr(\lambda w.\ \text{Sarah won} \mid \lambda w.\ \text{W is from group 31-100}) > 0.5 \wedge$$
$$\Pr(\lambda w.\ \text{Sarah won} \mid \lambda w.\ \text{W is from group 1-70 or 31-100}) \leq 0.5 \text{ etc.}$$

If the meaning computed in (63) holds of a situation, then replacing the domain of the NPI with any subdomain in (10) has a meaning that also holds of that same situation (at least on the assumption that NPIs only induce subdomain and scalar alternatives, e.g., Krifka 1995, Chierchia 2013, Crnič 2019): namely, note that in the sentence in which NPI *any* ranges over a smaller subdomain we, at most, lose an entailment pertaining to one of the two elements in the domain of *any*, that is, we lose one of the conjuncts in (63). The condition in (61) is accordingly satisfied.

The analysis of NPIs in these examples resembles the now-deprecated universal quantifier analysis of NPIs (e.g., Quine 1960, Lasnik 1972), on which *any* NPs are analyzed as a kind of universal quantifiers that must take scope above appropriate expressions. Importantly, though, in contrast to these earlier analyses, we are making no special assumptions about the nature of these expressions other than two uncontroversial ones: that they are indefinites, and that they are subject to a condition like (61). Moreover, our treatment above constitutes only one of the possible parses of the sentences under discussion (and a dispreferred one at that), and it cannot be applied in a host of other environments. Consider, for example, the simple conditional sentence in (64).

(64) If the winning ticket is from any of the three groupings of tickets, Sarah won.

We saw that if exceptional scope is assigned to the NPI in this sentence, it cannot be strengthened to a universal meaning (Sect. 4.1, §3). Accordingly, the parse in (65) violates the condition in (61) – the contribution of the NPI matches that of other indefinites in upward-monotone environments, and so the sentence is not downward-monotone with respect to its domain on this parse.

(65) #[exh$_{C'}$ [exh$_C$ [any of the three groupings]$_x$ [if W is from x] [S won]]]

The sentence may, hence, only be parsed with the NPI in the antecedent of the conditional. Exhaustification is necessary on this parse as well, as provided in (66) – recall, namely, our assumption that the conditionals have a non-monotone, closeness-based semantics (see Bar-Lev and Fox 2020, Sect. 7, for the prototypical derivation with disjunction).[6]

[6]The observed reading of the sentence is derived from (i) on a specific assumption about what alternatives are relevant

(66) [$\text{exh}_{C'}$ [exh_C [if W is from any of the salient groupings] [S won]]]

In summary: Indefinite NPIs may take the widest exceptional scope. Their acceptability in such cases depends on whether exhaustification can strengten the exceptional scope NPI indefinites so they convey universal meanings, as required by the NPI licensing condition. Since this obtains only in very specific circumstances, the availability of such construals has eluded detection.

A characterization of NPIs as inherently resistant to taking exceptional scope has been provided by Barker (2018). In relation to the usual cases, like the ones (64) and their ilk, the generalization is correct. Only in specific circumstance can the licensing condition on NPIs be satisfied by an indefinite taking exceptional scope. One class was studied in the preceding, another class is coming up next.

5.2 Exceptional intermediate scope

1. Indefinite NPIs can escape islands without taking widest scope. Let us begin by looking at a variant of Reinhart's (1997) well-worn examples in (67):

(67) a. [Context: A math textbook contains 500 difficult problems. Every math grad student is required to pick a problem and study every analysis that solves it. Tali studied every analysis that solves the four-color theorem. Zali studied every analysis that solves the Poincaré conjecture. But, as always, Gali is an exception.]

b. Gali DIDN'T study every analysis that solves ANY problem mentioned in the book.

The sentence has the plausible meaning that corresponds to there being no problem in the book such that Gali studied its every analysis; the *in situ* interpretation of the NPI would be a contextual tautology as no one is able to study every analysis of the problems in the book. The observed meaning can be derived from the strucure in (68), where the NPI takes intermediate scope, that is, scope above the universal quantifier and below negation. Note that no exhaustification is required in

(see Bar-Lev and Fox 2020 for a derivation without this assumption). In particular, we can derive the target universal inference for the sentence, which is required for NPI licensing, if we assume that the sister of the lower *exh* is not relevant at the lower level of exhaustification. This is a substantive assumption: for example, while it respects a condition on pruning in Crnič et al. 2015, a condition on pruning by Magri 2009 would have to be weakened to admit it from 'the sister of an *exh* operator having to be in the domain of that *exh* operator' to, say, 'the sister of an *exh* operator having to be a subconstituent of some alternative in the domain of some *exh* operator c-commanding it'.

this configuration in order for the licensing condition on *any NP*s to be satisfied.

(68) [neg [[any problem]$_x$ [[every analysis that solves x]$_z$ Gali studied x]]]

(69) $\neg\exists$x: problem x $\wedge$ $\forall$z: analysis that solves x $\rightarrow$ Gali studied z

2. NPIs are unacceptable in certain non-monotone environments, say, in the immediate scope of quantifiers like *between 2 and 5 analyses* at LF (e.g., Crnič 2014 for a recent discussion). Accordingly, the acceptability of the sentence in (70b) in the scenario in (70a) suggests that the NPI *any problem in the book* does not occur at LF in the relative clause in which it surfaces.

(70) a. [Context: Every student had to study between 2 and 5 analyses that solve some problem mentioned in the textbook. Tali studied between 2 and 5 analyses that solve the four-color theorem. Zali studied between 2 and 5 analyses that solve the Poincaré conjecture. But, as always, Gali is an exception.]

b. Gali DIDN'T study between 2 and 5 analyses that solve ANY problem in the book.

The sentence in (70b) may be assigned the structure in (71), where *any problem in the book* moves to an intermediate scope position. The meaning of the sentence is in (72).

(71) [neg [[any problem]$_x$ [[between 2 and 5 analyses that solves x]$_z$ Gali studied x]]]

(72) $\neg\exists$x: problem x $\wedge$ $2 \leq |\{$z | analysis z solves x $\wedge$ Gali studies z$\}| \leq 5$

3. Finally, Crnič and Buccola (2019) discuss the contrast between the sentences in (73), where the NPI *any French author* appears at surface form in the restrictor of a singular definite description. Strikingly, the sentence in (73b) can be used felicitously. For example, this may be the case, say, in a situation where, precedingly, one goes through a list of French authors paired with their salient books (Gali didn't read the book that Balzac wrote, Gali didn't read the book that Flaubert wrote, etc). The acceptability of (73b) is striking as singular definite descriptions are, on the one hand, known not to license NPIs due to their non-monotonicity or their joint Strawson upward- and downward-entailingness; see, e.g., Lahiri 1998, Gajewski and Hsieh 2014, Crnič 2019 for discussion) and, on the other hand, extraction out of their relative clauses tends to be impossible.

(73) a. #Gali read the book that any French author wrote.

b. Gali didn't read the book that any French author wrote.

Importantly for our purproses, the sentence in (73b) can only convey the reading on which the indefinite takes scope between negation and the singular definite description, that is, the reading on which the NPI is assigned exceptional intermediate scope. The pertinent structure and its interpretation are provided in (74)-(75):

(74) [neg [[any French author]$_x$ [Gali read the book$_z$ x wrote book z]]]

(75) $\neg\exists$x: French author x $\wedge$ Gali read ιz(book z $\wedge$ x wrote z)

It should be emphasized, however, that the presuppositional nature of singular definites imposes exacting conditions on the felicity of such sentences, causing this rescue strategy to be of limited value. For example, in setting up example (73b), we carefully identified single books by the different French authors in order to assure that the sentence can be used felicitously: for each salient French author, a single salient book that they wrote was mentioned. If you replace, say, the *any NP* with *ever,* which resists a restriction to salient temporal periods (Krifka 1995), we may obtain contextually contradictory presuppositions (for each moment or interval of time, there would have to be a unique existing element picked out by the restrictor of the definite).

In summary: Indefinite NPIs may take intermediate exceptional scope in appropriate contexts. Their acceptability may in these cases follow simply from them occurring within a constituent that is downward-monotone with respect to them, hence exhaustification may be unnecessary.

6 Conclusion

Our starting point was the apparently paradoxical behavior of disjunction and indefinites: they can convey conjunctive and universal meanings, respectively, without apparently conveying their standard disjunctive and existential meanings. We showed that this behavior follows from the interaction of two properties of disjunction and indefinites: their exceptional scope taking and their ability to be strengthened in certain configurations. Thus, the observed behavior was shown to be neither paradoxical nor to require any new theoretical assumptions. While we fleshed out our proposal in terms of movement and exhaustification, combinations of other mechanisms that derive the two properties of disjunction and indefinites may allow for analogous derivations as well.

Finally, this behavior is not restricted to regular indefinites, but can be found with indefinite

NPIs as well. This means that indefinite NPIs share yet another feature with other indefinites – the ability to take exceptional scope. This is usually veiled by the requirement that they satisfy the NPI licensing condition, a condition that can be satisfied on their exceptional scope construals only if these are in a downward-monotone environment, or if their existential import is strengthened to universal one.

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